Intransitive Classification and Choice: A Review

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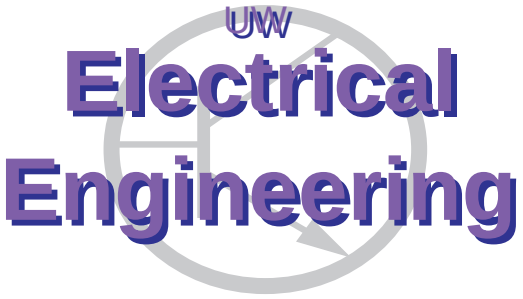

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Abstract

A classifier is a machine that takes a description of an object, and then labels that object as being of a particular category. Classifiers have a rich history in the fields of statistics, artificial intelligence, and machine learning. This paper gives a brief background in statistical pattern recognition, multi-class classification, transitive games, and presents a brief history of intransitive preferences in a variety of fields.

Project Summary: Intransitive Classification and Choice

1 Introduction

In today's world, one of the crucial problems facing humankind is that of building machines capable of automatically making intelligent decisions in potentially adverse or uncooperative environments. The applications for such an ability span across many different fields, ranging from commercial manufacturing and automated robotics control, to defense related machinery, space exploration and satellite navigation, elections, intelligent human-computer interfaces such as speech and vision systems, security and biometrics, and to assistive devices for the handicapped and the aged — and this is just a few. In all cases, the goal is to produce a machine that appears to act both "robustly" and "intelligently." While a difficult term to define precisely, to act "intelligently" might mean to act in a way such that it appears a human being, after having undergone careful deliberation, has made a conscious decision to achieve some pre-specified and precisely stated goal or plan.

There are many different forms of intelligent decision making. In sequential decisions making, for example, the set of decisions are made one after another in a multi-stage process. A decision made at a given time can have an affect and possibly limit decisions made at some time in the future. A decision is typically made based on (sometimes partial) knowledge of the world, and based on optimizing an expected reward function over some time period spanning into the future. Markov decision processes [139, 147, 100, 185, 161] are one way to model such a sequential decision maker, where knowledge of the world at any given time is either complete or only partially known [112, 119, 144, 158].

Another style of decision making is well captured by the notion of a classifier. In this proposal, a *classifier* means a machine that takes as input an object encoded in some way, and based on that encoding makes a decision, such as deciding the class, category, or type of that object, and might also include rank information as well. In the simplest of cases, a decision is made once and the classifier must accept the consequence of having made that decision. No future decision can be considered as a weight or bias to influence current decisions, and no past set of decisions or function of a world state is available to help the classifier in making its decisions. Therefore, a classifier can be seen as a stateless machine with an input representation of an object or environment and output which corresponds to a decision to be made. Of course, one might augment the input to include both the state of the environment, the machine, and past decisions, and thereby produce sequential decision procedures, as in a recurrent neural network [86].

Such classifiers are a crucial part of many intelligent decision making machines. This includes statistical pattern recognition [50, 51], discriminant analysis [126], and regression [72, 84]. These, in turn, encompass a wide variety of applications such as speech [203], face [197], and hand-writing recognition[113], texture identification, land cover classification [152], security-based biometrics such as retinal and iris scanning, finger-print recognition, voice identification, speaker verification, speaker [151] and language identification [105], and so on. The list is enormous.

A critical research problem in all such systems is to discover methods that will reduce errors with only a negligible increase in computational and parameter complexity. It is often the case that an error penalty can be extremely large, so any methodology that is capable of producing an appreciable error reduction can achieve huge cost savings. Indeed, much theoretical and practical research has recently occurred to produce better performing classifiers [126, 193, 51, 84].

2 Review

This section gives a brief background in statistical pattern recognition, multi-class classification, transitive games, and presents a brief history of intransitive preferences [101, 122, 171, 182, 134, 3, 189, 67, 172, 125, 60, 62, 65, 7, 13, 120, 163, 184, 2, 88, 132, 188, 34, 155, 154, 196]. A small part of the material presented here was initially presented at [19, 91].

2.1 Statistical Pattern Classification

The foundation behind all pattern classification involves the identification of the *class* or category of some unknown object x. There are a set of K classes $C_1, \ldots, C_K$ and the object is presumably one of those classes. The *input* x typically (but not necessarily) is a vector with real or integer components. A pair (x, y) is called an *example* or *data point*, and y is called the *label* of x. The true association between inputs and labels can be deterministic or stochastic.

A *classifier* is a system that takes an input x and outputs $\hat{y}(x)$, a guess of its true class label y. The theory of pattern classification [50, 51, 126, 193] shows that even in the case of a stochastic dependence, it is optimal for a classifier to output a single y as response to an x. Therefore, we focus on classifiers that take x as input and output a single label y from the set $\{1, 2, \ldots K\}$.

A classifier is *trained* on a particular problem (e.g., digit recognition, identifying tumors in MRI images, identifying the category of a news article) by using a set of *examples* $\mathcal{D} = \{(x^n, y^n)\}_n$. There are many different types of classifiers, including logistic regression [126], multi-logistic regression [126], multi-layered perceptrons [28, 153], support vector machines (SVMs) [193, 170], classification trees [31] and there are many ways to train once a model family has been chosen [6, 97, 98, 199, 54, 53, 55].

Generative vs discriminative classifiers: It is possible to view classification purely statistically, where each class C_i is a stochastic process generating vectors x according to the distribution $p(x|i)$. The prior probability of the next example coming from class C_i is $P(i)$. Therefore, by Bayes rule, one can compute the posterior probability given x of label $y = i$ as $P(i|x) = P(i)p(x|i)/\sum_j P(j)p(x|j)$ The classifier then outputs $\hat{y}(x) = \text{argmax}_i P(i|x)$.

The models $p(x|i)$ are called *generative* and classifiers employing them are called *generative classifiers* (Naive Bayes being one example). Thus, a generative classifier models explicitly the process that generates data from each class. By contrast, a *discriminative* classifier (e.g., nearest neighbors, SVM, perceptron) focuses on modeling decision boundaries between classes. The latter has the advantage of utilizing information optimally; however, generative classifiers are important especially in tasks involving complex inputs because they allow a user a more flexible way of entering prior knowledge about the problem through the generative model. Moreover, generative classifiers can be trained [54, 53, 55] and induced [22, 26] discriminatively.

Binary vs multiway classification: A classification problem with $K = 2$ classes is a *binary* problem, while when $K > 2$ we say that we have a K*-way* or a *multiway* classification task. Binary classifiers (BCs) form a group apart for two major reasons: First, being simpler BCs can be more easily analyzed. In fact, one can cast any binary classification problem in the form $\hat{y} = \text{sign} g(x) \in \{\pm 1\}$, where g is a real-valued function called *discriminant*. For example, binary classification with generative models is expressed as $\hat{y}(x) = \text{sign} \log \{P(y = +1)p(x|y = +1)/P(y = -1)p(x|y = -1)\}$. We also define the log *likelihood ratio* between two classes i and j as $L_{ij}(x) = \log p(x|C_i)/p(x|C_j)$, and a general real-valued "preference" $S_{ij}(x)$ for class i over class j, either of which can be used as a binary classifier between class i and j.

While some classifiers can directly incorporate any number of classes (nearest neighbors, multi-layer perceptrons, generative classifiers, decision trees), others are naturally defined only for the $K = 2$ case (SVMs, decision stumps, linear perceptrons). Significant work has gone into constructing K-way classifiers using several binary classifiers as building blocks (see below). This direction is a main objective of our proposal, so in the next section we survey relevant work.

2.2 K-way classification as coding

K-way classifiers can be constructed using binary classifiers as building blocks. This is an active and important subfield since: 1) some of the best overall performing classifiers (voted perceptron, SVM) are naturally defined as binary classifiers; and 2) theoretical guarantees for K-way classifiers can sometimes be derived from the existing results for the binary ones.

One of the most popular ways of *binarizing* a K-class problem is the *all-pairs* (or *pairwise coupling* or *round-robin*) approach [71, 85, 146, 107, 41, 42, 133, 123, 124, 204] described presently. Let $\mathcal{D}_k$ represent all the examples representing class k. Binarization proceeds as follows: in the **learning stage**, for each pair of classes (k, l) train binary classifier g_{kl} on only the $\mathcal{D}_k \cup \mathcal{D}_l$ subset of the training set. In the **classification stage**, we label an unknown input x as $\hat{y}(x) = \text{sign}[h(\bar{g}(x))]$ where $\bar{g}(x) = (\, g_{kl}(x)\,)_{k<l}$ is the vector containing the outputs of all binary classifiers and the *decoding* function h is some function of the $M = K(K-1)/2$ variables in $\bar{g}$. In this paradigm, the BCs are treated as black boxes and the challenge is to design and analyze the function h.

The *one-vs-all* method, often used with neural networks [28, 153], constructs one classifier g_k to discriminate between examples in class k and all other classes. A possible problem with this method is that more than one $g_k(x)$ can be positive for an input x (note that *base classifiers* g_k or g_{kl} can have either discrete ± 1 or continuous real-valued outputs). Therefore, the label is typically calculated as $\hat{y}(x) = \text{argmax}_k\, g_k(x)$.

A powerful approach, the *error correcting code* (ECC) method [47] and the classifiers inspired by them are best described in the unified framework proposed by [1]. Define a $K \times M$ *code matrix* H, where K is the number of classes and M is the number of binary classifiers used, as follows: Column m of H is a vector $H_{:m}$ containing the

values $\{0, +1, -1\}$. Denote by C_{m+} (resp. C_{m-}) the set of class indices represented by a "$+1$" (resp. "-1") in $H_{:m}$. Classifier g_m is trained to discriminate between the classes in C_{m+} vs. the classes in C_{m-} on the corresponding subsets of training data $\bigcup_{k \in (C_{m+} \cup C_{m-})} \mathcal{D}_k$.

For example, in the one-vs-all approach, $M = K$, $\forall m \ H_{mm} = 1$ and $H_{km} = -1$ for $k \neq m$. In pairwise coupling, $M = K(K-1)/2$, the variable m indexes the pairs $(k, l),\ 1 \leq k < l \leq K$ and for an m corresponding to (k, l), we have $H_{km} = +1,\ H_{lm} = -1,\ H_{k'm} = 0$ for all $k' \neq k, l$. In the ECC approach [47], H is the matrix of an error correcting code (thus typically $\log_2 K \leq M \leq K$). This ensures that the rows of H (each row representing one class) can be well separated. The label of an unknown input x represents the row $H_{k:}$ of H that is closest in Hamming distance to the vector $\bar{g}(x) = [g_1(x)\ g_2(x)\ \ldots\ g_m(x)]$, i.e., $\hat{y}(x) = \operatorname{argmin}_k d_H(\bar{g}(x),\ H_{k:})$. In general, classification is seen as "encoding" an input x by using M functions g_m, followed by a phase where the vector $\bar{g}(x)$ is "decoded" according to the code matrix H. In the original ECC paper, the classifiers g_m output ± 1. Further work of [39] showed that constructing a coding matrix with ± 1 entries given the M base classifiers is NP-hard and generalized ECC to code matrices with real-entries and real-valued g_m classifiers. The papers of [44, 159, 1] explored the optimal design and learning of H and g.

Boosting is a method of constructing a classifier $\hat{y}$ by sequentially training the blackbox classifiers $g_1,\ g_2, \ldots\ g_m, \ldots$ on differently weighted versions of the same training set $\mathcal{D}$. If one assimilates the columns of H above with weighting functions over the data, then boosting can be seen as part of the same encoding/decoding family of algorithms. Boosting was originally designed for binary classification [177] but it was soon extended in various ways to multiway classification [169, 70, 177, 82, 166].

2.3 Preferences, Tournaments, Transitivity, and Likelihood Ratios

The theory of transitive preferences and tournament games is well studied [116, 184, 132]. In general, there are K objects, $A_k \quad k = 1, \ldots, K$ and a (resp. strict) binary preference relation $\succeq$ (resp. $\succ$) which encodes the preference patterns of an organism for each pair of objects. For example, if $A_i \succeq A_j$ (resp. $A_i \succ A_j$) then A_i is either preferable or indifferent to A_j (resp. A_i is strictly preferable to A_j).

It is common to encode preferences using utility functions $u(A_i)$ [116] that map each object to a real number such that standard comparison operators reflect the exact same preferences as does the $\succ$ relation. For example, if $u(\cdot)$ is such a utility function, then $u(A_i) \geq u(A_j)$ if and only if $A_i \succeq A_j$. It is often noted that utilities are not "true numbers", in that there are many possible values for each utility function which encode exactly the same preferences. Therefore, meaning should not be given to the absolute values of and relative differences between utility values (i.e., use of utilities is insensitive to monotone transformations).

Preference relations and utilities can be used to encode tournament style games. In such a game, there are K players. Pairs of players compete with each other, and there is either a winner or looser (or a tie). If A_i and A_j are players, and if $A_i \succ A_j$, then A_i has won over A_j. Tournaments can be formally defined as a binary relation over a set of objects [132] and can be used to encode tournament-style games such as tennis, football, world-cup soccer, and so on.

It is often assumed that preference relations are transitive. This means that if $A_i \succeq A_j$ and $A_j \succeq A_k$, then $A_i \succeq A_k$. Transitive relations go hand-in-hand with utilities; a set of single real-valued utility functions can be defined that encodes preferences iff these preferences are transitive. In other words, strict transitivity means that the objects being compared can be placed into rank order, where an object's relative rank is used to determine preference. Once objects have a rank, they have a strictly sequential ordering and transitivity results. Transitive tournament style games can be encoded using directed acyclic graphs, where a directed edge points from a looser to a winner. In such a graph, the final winner has no outward pointing edges.

Note that many standard statistical pattern classification methods can be cast in terms of transitive preferences and transitive tournament-style games (this idea was originally presented by the PIs in [19]). The function $L_{ij}(x)$ then codes, in the context of an object represented by x, a preference relationship between either class i and class j as being the category for object x. That preference relation is $C_i \succ C_j \iff L_{ij}(x) > 0$. Given a particular ordering of the classes, $\{C_1, C_2, ..., C_K\}$, a game strategy proceeds by evaluating $L_{C_1 C_2}$ which if positive means that C_1 has "won" so it is followed by evaluating $L_{C_1 C_3}$. If C_1 lost, the procedure is followed instead by evaluating L_{C_3, C_2}. This procedure continues until a final winner is found. If posterior probabilities $p(C_j|x)$ are used rather than likelihoods, the final winner is the same as what is decided by Bayes decision rule. Neither a class order permutation nor a monotone transformation on the probabilities affects the ultimate winner. This is because underlying the relationships are utility functions $u(C_i) = \log p(x|C_i)$ corresponding to the likelihoods (or log posteriors), thus implying transitivity.

2.4 Intransitivity in Decision Making

A given preference relation need not be transitive. An *intransitive relation* arises when there is at least one instance of three objects A_i, A_j, and A_k such that $A_i \succeq A_j$, $A_j \succeq A_k$ but $A_k \succeq A_i$. Perhaps the simplest and most widely known intransitive game is that of rock-paper-scissors where rock wins over (crushes) scissors who wins over (cuts) paper who wins over (covers) rock, thereby creating a loop. Intransitive games can be depicted by directed graphs that posses directed cycles.

There has been much debate in the past regarding whether it is logical for intransitive preferences to exist in natural organisms [116, 122, 3, 2]. Transitivity, for example, is sometimes seen as a consequence of "rational thought", as any rational being would not permit itself to have intransitive preferences. One such argument against intransitivity makes this case quite clearly. Suppose we encounter a person who prefers apples over bananas, bananas over oranges, and oranges over apples. Suppose also that the preferences are strict, such that there is no pairing in which there is an indifference between any two fruit. When such a person is in possession of an apple, he would be willing to exchange it plus a small sum of money for a banana. With a banana, he would exchange it plus money for an orange, and given an orange he would exchange it plus money for an apple. The cycle would then continue indefinitely, leading to what has been called a "money pump." [2, 163, 40]. Some have claimed that such preferences would indicate irrational judgment by the person since economic survival requires that individuals behave as if they had transitive preferences. Others have claimed that intransitive relations (indicating friendship between individuals) are inherently unstable (short lived) [57]. In fact, there are many such discussions against intransitive preferences, but most of these arguments are quite specialized and philosophical in nature. In recent years most of them have fallen out of favor [69, 120, 2, 173], and intransitive preferences are now even seen as being sophisticated [163].

Regardless of the rationality of transitivity, intransitive relations represent many real-world phenomena such as sports tournaments (team A wins over team B, who wins over team C, who then wins over team A). It has also been demonstrated that intransitivity can represent preference patterns in both humans [189, 114, 115, 205, 30] and non-human animals [118, 196, 176, 150, 89, 198, 136, 8].

Intransitive preferences may arise from one of a number of causes. First, preferences can potentially be formed by the use of non-optimal "heuristics and biases" [190], where a decision is formed that can have a high probability of errors — intransitivity results from these errors. On the other hand, intransitive preferences can arise by ever changing preferences, where the act of choosing itself determines a preference [34]. Alternatively, intransitive preferences can result from ecologically advantageous "fast and frugal" or simple heuristics which are, essentially, both computationally cheap but perform well in nature [80, 68]. These heuristics fall under the category of satisficing procedures [81, 80], an instance of "bounded rationality" [181, 180], where the minimum satisfactory condition or outcome is chosen, and where past experience is used to gauge which choices are those that satisfice. Yet a third viewpoint espouses the view that intransitivities can result from context dependences [88]. In this case, the preference depends on a (sometimes hidden or unknown) context, and it is only by not controlling for these contexts do intransitivities arise.

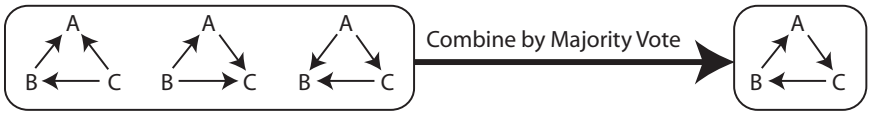

Figure 1: The combination by majority vote of three individuals with transitive preferences (left three graphs) into intransitive preferences (right most graph). Each directed graph depicts pairwise preferences, where an arrow points to the more preferred from the less preferred.

One particularly compelling explanation in favor of intransitivity arises from preference combination in majority voting schemes [3, 125, 163, 164]. In particular, it is possible to combine pairwise preferences of three agents using a majority vote for each pair, and arrive at intransitive preferences. This is shown by the graphs in Figure 1. This is an important example as it suggests that multiple individuals, each of which exhibit purely "transitive" preferences, will exhibit intransitivity when their preferences are jointly and democratically represented (i.e., what has been called "social choice"). Moreover, Arrow's [3] famous theorem shows that, under several reasonable assumptions, no system of elections with more than two candidates is "rational" in this sense except for a dictatorship [3, 163]. This implies that intransitivity can be explained by the presence of multiple "hidden" agents whose preferences are combined in some way An individual's intransitive preferences might also be explained by multi-agent descriptions of mind [131].

Another compelling example of mathematical intransitivity is the notion of *non-transitive dice*, where given three fair cubic 6-faced dice A, B, and C, it is possible to label the faces such that $P(A > B) > 1/2$, $P(B > C) > 1/2$,

and $P(C > A) > 1/2$ [77, 78, 187, 96]. While potentially appearing paradoxical, clearly this example demonstrates that intransitivity can easily arise via a probabilistic model.

Intransitivity also occurs in coevolution [58, 137, 33, 95], a branch of evolutionary science where one population evolves directly in response to the evolution of other (competitive) populations. Coevolutionary algorithms are often used for the evaluation of populations in pairwise comparisons with these other populations, but the existence of intransitivity in such evaluations can often lead to cycles, where previously visited evaluative states are re-visited over and over.

In fact, there are a number of mathematical and statistical models of preference that can result in intransitivity. These include multi-attribute or multi-dimensional preference models [62] including a lexicographic semi-order [189], non-transitive additive models [62], and additive difference models [189]. In the field of consumer demand theory, it is also possible to prove the existence of competitive equilibrium without assuming transitivity [183] and that such intransitive orderings can be represented by a continuous numerical function [175]. Many other models are also possible.

Without a doubt, at this point intransitive preferences have been demonstrated to exist in both natural organisms and mathematical models. Therefore, any understanding of natural decision making should strive to understand the reasons behind and the modeling of such intransitive preferences. Proposed below is methodology that represents multi-class classification as an intransitive game and that can exploit these preferences both for classification and understanding group choice. As will be seen, research is then proposed to represent and exploit this intransitivity in many novel ways.

3 Classification As Intransitive Games

Classification can be seen as an intransitive game, where intransitivity can occur either by "corrected" likelihood-ratio classifiers [19, 91] (Section 3.1), or by a set of binary classifiers used to construct a multiway classifier. We use the notation $S_{ij}(x)$ to indicate a preference for class i over class j given unknown object x. With K classes, there are at most $K(K-1)/2$ such preferences.

3.1 Intransitivity via Augmented Likelihood Ratio

In previous work [19, 91], it was shown how significant error rate reductions can be obtained by making corrections to the *approximate* likelihood ratio defined as $\hat{L}_{ij}(x) \stackrel{\Delta}{=} \log \frac{\hat{p}(x|c_i)}{\hat{p}(x|c_j)}$, so that it is a better estimation of the true likelihood ratio $L_{ij}(x)$. Using this, a preference relation $S_{ij}(x)$ was obtained by likelihood ratio augmentation. In general, there will be error between $\hat{L}_{ij}(x)$ and $L_{ij}(x)$ for each x. The goal is to find the best x-independent correction term α_{ij} that minimizes the error between the corrected likelihood ratio $f(\hat{L}_{ij}(x) + \alpha_{ij})$ and the true likelihood ratio $f(L_{ij}(x))$, where both ratios are first processed by a function $f(\cdot)$ (see below). Note that for simplicity, the techniques as presented here are introduced using likelihoods $p(x|C_k)$ and their estimations $\hat{p}(x|C_k)$, but the methodology may use any of the discriminant functions, BCs, and preference relations as described below, in Sections 2.1, 2.3, and in [91].

The goal is to find the α_{ij} that minimizes the error, defined as the expected squared difference between the corrected approximation and the true term modulo an importance function $f()$, or:

$$J(\alpha_{ij}) = \frac{1}{2} \int \left(f(\hat{L}_{ij}(x) + \alpha_{ij}) - f(L_{ij}(x)) \right)^2 p(x) dx, \tag{1}$$

the minimization of which leads to the optimal alphas $\alpha^*_{ij} = \text{argmin}_{\alpha_{ij}} J(\alpha_{ij})$.

Given these best α_{ij} coefficients, a new preference relation can be defined as $S_{ij}(x) = \hat{L}_{ij}(x) + \alpha_{ij}$, so that $C_i \succ C_j \iff S_{ij}(x) > 0$. A key issue is that $S_{ij}(x)$ can become an intransitive preference relation. First, if the correction terms α_{ij} are perfect, so that $S_{ij}(x) = L_{ij}(x)$, then the true likelihood ratios are obtained, and therefore $S_{ij}(x)$ would be transitive since likelihood-ratio based preferences are always transitive. It is unlikely, however, that prefect correction occurs or is possible. Therefore, since the α_{ij} coefficients are a function of both i and j and there is no longer an underlying utility to rank-order the categories, intransitivity can result. Note that due to the potential intransitivity, the resulting preference relations $S_{ij}(x)$ have escaped from the family of possible models representable purely by a likelihood ratio.

The resulting preference relation, itself, was seen to significantly improve classification accuracy [19], and when seen as a intransitive game, further improvements were found. Specifically, in our previous work [19], the PIs optimized this function using $f(a) = a$ (the identity function), and obtained, under certain assumptions, that the best α_{ij} is a *difference* between two Kullback-Leibler divergence terms.

This corrected term when used as a preference relationship produces significant improvements in classification accuracy over the baseline likelihood ratio on an isolated-word speech recognition task, where the likelihood functions were hidden Markov models (HMMs) [148, 16]. We demonstrated empirically that S_{ij} is in many cases indeed intransitive. More importantly, there were several beneficial strategies for exploiting such intransitivity. These included playing a multi-player tournament a number of times, each with a different random permutation of players. The final class chosen was the most frequent winner among the tournaments, additionally improving classification accuracy. Further, we found that the degree of intransitivity, defined via detected cycles in the winner-looser directed graph, is a good indicator of error: if a large number of different cycles is found, the chance of error occurs is large. Therefore, the degree of intransitivity appears to be a useful *confidence* or *inverse robustness* measure.

In subsequent work [91], it was demonstrated that the methodology presented above is general beyond just isolated-word speech recognition, and showed improvements on the UCI machine learning data sets [29, 87]. This was done using both $f(a) = a$ as above, and $f(a) = \text{sign}(a)$ and various of its continuous differentiable approximations. The sign function case was called "necessary corrections" since it asks for the least change to $\hat{L}_{ij}(x)$ to achieve sign equality with $\hat{L}_{ij}(x)$, the minimum that is necessary to obtain perfect classification. Moreover, improvements were found for two baseline classifiers: 1) multi-layer perceptrons trained using a KL-divergence error function [28, 153] where the term corrected was a ratio of posterior probabilities; and 2) Naive Bayes classifiers [73, 49, 108]. In both cases, there were significant improvements over the baseline.

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