Static Channel Assignment in Multi-radio Multi-Channel 802.11 Wireless Mesh Networks: Issues, Metrics and Algorithms

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***Abstract*— The combination of multiple radio nodes in conjunction with a suitably structured multi-hop or mesh architecture has the potential to solve some of the key limitations of present day wireless access networks that are based on single-radio nodes. This paper addresses the static channel assignment problem for multi-channel multi-radio static wireless mesh networks. We present four metrics based on which mesh channel assignments can be obtained. In particular, we focus on minimization of the average and maximum collision domain sizes and show that these problems are closely related to problems in combinatorial optimization such as MAX k-CUT and MIN k-PARTITION. We also present heuristic algorithms for solving the channel assignment problems using the above two metrics.**

I. Introduction

Traditional multi-hop wireless networks (studied since the 70's as packet radio networks) have almost exclusively comprised of single radio nodes. It is well-known that in such networks, the end-to-end throughput on a route drops as the number of hops increase. A primary reason is due to the fact that a single wireless transceiver operates in half-duplex mode, $i.e.$, it cannot transmit and receive simultaneously. An incoming frame must therefore be received fully before the node can switch from receive mode to transmit mode. Consequently, for a linear chain topology of n nodes where only one transmission is allowed at a time in the network[1], the per-node throughput is on the order of $O\left(\frac{1}{n}\right)$ for a CSMA/CA type MAC. More generally, it has been shown by Gupta and Kumar [1] that the per-node throughput of an ad-hoc network scales asymptotically as $O\left(\frac{1}{\sqrt{n}}\right)$, if the source-destination pairs are chosen randomly.

Multiple radios greatly increase the potential for enhanced channel selection and route formation while the mesh allows more fine-grained interference management and power control. There are several interesting research issues in the context of multi-radio, multi-channel wireless mesh networks (WMN); finding the *optimum channel assignment* for a given number of radios per node and a given number of orthogonal channels is the objective of this work. It should be noted, however, that use of multiple radios to exploit the availability of multiple non-overlapping channels is not *the* silver bullet for improving multi-hop throughput in wireless networks. Other approaches which have been researched include use of directional antennas which reduces the interference area around a transmitting node [2] and improved MAC protocols [3]. It is likely that a suitable combination of these approaches would lead to next generation multi-hop network design. However, in our opinion, outfitting each node with multiple radios is probably the most cost-efficient solution which does not require expensive new hardware or complex modifications to the existing MAC protocols. While mutual interference among the multiple radios (NIC) on a node could limit the degree of actual improvement, it is expected that advanced EMI protection and device integration techniques would mitigate the mutual RF interference considerably.

II. Related Work

There are a number of common issues involved in traditional multi-hop wireless networks. These, as was noted in [4] and [5], include efficient methods for sharing the common radio channel, network connectivity, network capacity, and methods for managing and controlling the distributed network. A particular issue that is of interest to us is the channel assignment problem in multi-hop wireless networks with a single radio. This issue has been subject to several studies in the literature. Early work by Cidon and Sidi [6] presented a distributed dynamic channel assignment algorithm that is suitable for shared channel multi-hop networks.

A natural way to increase network capacity and utilization is by exploiting the use of multiple channel and channel reuse opportunities. Several studies on the subject of *multi channel* multi-hop wireless networks have been the main subject of research in recent years. In [7], [8], [9], [10], for example, MAC protocols based on modification of IEEE 802.11 were proposed for utilizing multiple channels. In particular, Jain *et al* [7] propose a protocol that selects channels dynamically and employs the notion of "soft" channel reservation. This reservation based scheme, which was later extended in [8], gives preference to the channel that was used for the last successful transmission. So and Vaidya [9] propose a MAC protocol which enables hosts to dynamically negotiate channels such that multiple communication can take place in the

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[1]This is justified when the carrier sensing range is sufficiently large or the network size is sufficiently small.

This work was supported by a grant from Intel Research Council.

same region simultaneously, each in different channel. The proposed scheme requires only a single transceiver for each host. They later extend their study in [10] to propose a routing protocol for multi-channel multi-hop wireless networks with a *single* interface that finds routes and assigns channels to balance load among channels while maintaining connectivity.

A few approaches to the routing and channel assignment problems in multi-hop multi-radio mesh networks have been proposed [11], [12], [13]. Kyasanur and Vaidya [11] studied the multi-radio mesh network under the assumption that the network has the ability to switch an interface from one channel to another dynamically. They present a distributed interface assignment strategy that accounts for the cost of interface switching and does not make any assumptions on the traffic characteristics. Their routing strategy selects routes which have low switching and diversity cost taking into account the global resource usage to maximize the network utilization and allows the nodes to communicate without any specialized coordination algorithm. Raniwala *et al* [12], [13] propose a centralized load-aware joint channel assignment and routing algorithm, which is constructed with a multiple spanning tree-based load balancing routing algorithm that can adapted to traffic load dynamically. They demonstrate the dependency of the channel assignment on the load of each virtual link, which in turns depends on routing. They also show that the problem of channel assignment is NP-hard.

III. Network Model and Assumptions

We consider an N-node wireless mesh network in which all the nodes are stationary. We will assume that the nodes run a mesh MAC layer which allows them to dynamically change the channel to which each of their radios is tuned. Several such protocols have been proposed in the literature ([14], [15]), including as submissions to the ongoing standardization effort within IEEE 802.11 by Task Group 'S' on mesh networking [16]. The need for a mesh MAC protocol is the following. Suppose that there are F available orthogonal channels, and that each node has R radios, where $R < F$. The current 802.11 standard does not specify a mechanism for nodes to switch the channel to which a radio is tuned on a per-packet basis. This effectively means that if a node wishes to communicate with multiple neighbors using the same radio, it must communicate with all those neighbors on the same channel. Stated differently, a node is limited to using only R out of the F channels to communicate with its neighbors. The use of a mesh MAC protocol allows a node to switch to a different channel for each neighbor; $i.e.$, a node with k neighbors can use up to $\min(k, F)$ channels to communicate with its neighbors simultaneously, thereby allowing for greater channel diversity in the network.

Although a mesh MAC will typically allow neighboring nodes to choose the channel on which they will communicate on a per-packet or per-packet-burst basis (for 802.11e), we will only consider the case where a given pair of neighbors always uses the same channel to communicate. In this sense, although nodes dynamically switch their radios to different channels, the channel assignment itself is *static*.

IV. Channel allocation in WMN's with multiple radios and multiple channels

In this paper, we consider the static channel assignment problem on a network of N nodes. The network is allowed to be heterogeneous in the sense that all nodes are not required to have the same number of radio interfaces. We now look at the interference pattern in an 802.11 wireless network under the assumption that all nodes in the mesh employ the RTS/CTS mechanism to combat the hidden terminal problem before actual data transmission. When a single channel is available (which is what the IEEE 802.11 MAC protocol is designed for), after a successful RTS/CTS exchange between a pair of nodes, no node within virtual carrier sense range of the transmitter and receiver can communicate for the duration of the subsequent data packet. We will refer to the set of edges which must remain silent when edge e is active as the *total interference set* of edge e.

When multiple channels are available, we define the *co-channel interference set* of an edge e which is assigned channel f as the subset of its total interference set which have also been assigned channel f. We show in this paper that through intelligent channel assignment, it is possible to reduce the interference domain sizes significantly, compared to the single channel case. Intuitively, it is clear that minimizing the interference domain sizes have the effect of enhancing simultaneous transmissions in the network.

We now turn to the issue of choosing an appropriate metric for static channel assignment in WMN's. Typically, there will be many feasible channel assignments and we would therefore like an optimality criterion that allows us to pick one of these channel assignments. Given a set of available orthogonal channels, the goal of a static assignment scheme should be to use the channels as "best" as possible, thereby directly affecting the performance of a network. Some metrics which are suitable for static channel assignment are listed below. All of these attempt to increase the overall network performance by allowing more simultaneous transmissions, either directly (*Problem P*-1) or indirectly (*Problems P*-2 and *P*-3).

- *Problem P*-1: Direct maximization of the number of possible simultaneous transmissions in the network. Intuitively, such an assignment should maximize the 1-hop or link layer throughput in the network *in worst case traffic*; $i.e.$, when the traffic profile is such that there is simultaneous contending traffic on all links in the network. However, this may not guarantee maximum network layer throughput (an end-to-end metric), which is a dynamic criterion and depends on the real time traffic conditions in the network. Two different integer linear programming (ILP) models, possibly with different polyhedral properties, were suggested by Das *et al* in [17] for solving problem *P*-1 optimally.
- *Problem P*-2: Minimization of the average size (cardinality) of a co-channel interference set. This metric is analogous to the "minimization of the average transmitter power" criterion used for topology optimization in wireless networks.
- *Problem P*-3: Minimization of the maximum size of any

co-channel interference set, which is analogous to the "minimization of the maximum transmitter power" criterion used for topology optimization in wireless networks. This metric was also considered by Marina and Das [18]. For irregular networks which have only a few edges with potentially large co-channel interference sets, this might be a better optimization criterion than the metric discussed above.

In addition to the above metrics, *channel diversity*, defined as the difference between the maximum ($MAXUSAGE$) and minimum ($MINUSAGE$) number of times any channel is used,

$$\text{channel diversity} = MAXUSAGE \; - \; MINUSAGE \quad (1)$$

is an important criterion for channel assignment. However, simply ensuring a perfectly diverse assignment (channel diversity = 0) may not affect the simultaneous transmission capability of a network. We will therefore use it as a secondary criterion in conjunction with the other metrics discussed above. Note that the above definition of channel diversity is slightly counterintuitive since an assignment is in fact "more diverse" for smaller values of the r.h.s of (1).

In this paper, we focus on problems *P*-2 and *P*-3 and show that these are closely related to the MAX k-CUT problem[2] and its dual, the MIN k-PARTITION problem, which are defined below. Both these problems are known to be NP-hard. We also discuss heuristics based on an existing algorithm for the MAX k-CUT.

Definition 1 (MAX k-CUT): Given a graph $G = (V, E)$ and a positive integer k, find a partition of V into k clusters such that the number of inter-cluster edges (edges which have their endpoints in two different clusters) is maximized.

Definition 2 (MIN k-PARTITION): Given a graph $G = (V, E)$ and a positive integer k, find a partition of V into k clusters such that the number of intra-cluster edges (edges which have their endpoints in the same cluster) is minimized.

Note that the number of intra-cluster edges in Definition 2 is equal to half[3] the sum of the *indegrees* of the nodes. Given a node i in cluster f, the indegree of node i, $\delta_{i(f)}$, is equal to the number of intra-cluster edges incident on i in the induced subgraph G_f.

V. Minimization of the Average Size of a Co-Channel Interference Set

In this section, we first consider minimization of the average size of a co-channel interference set (*Problem P*-2). Subsequently, we extend it to the case when the maximum (or bottleneck) size of a co-channel interference set is to be minimized (*Problem P*-3). It is important to note that minimizing the average size may not minimize the maximum size, or *vice versa*.

[2]The MAX k-CUT problem is a generalization of the well studied MAX CUT problem for $k = 2$.

[3]The factor $1/2$ is due to the fact that each edge is counted twice when the node indegrees are computed.

Given that a particular (transmitter, receiver) pair is communicating on channel f, the total interference set defined in the previous section (for the single channel case) can be regarded as the set of *potentially interfering edges*; these edges can only interfere with the ongoing transmission if they are also assigned to the same channel.

Definition 3: For any bidirected edge $e = (i \leftrightarrow j) \in \mathcal{E}$, where $\mathcal{E}$ is the set of all bidirected edges in the network, the set of its potentially interfering edges, denoted by $IE(e)$, is given by:

$$\begin{aligned} IE(e) = &\text{ all edges incident on } \{ne(i) \setminus j\} \;\cup \\ &\text{ all edges incident on } \{(ne(j) \setminus i\} \end{aligned} \quad (2)$$

where $ne(i)$ is the set of neighbors of node i and '$\cup$' denotes the *union* operator.

Note that alternate definitions of potentially interfering edges (for example, SINR based) are possible and can easily be accommodated within the framework of this paper. We next define a *link interference matrix* based on the sets of potentially interfering edges.

Definition 4: Given an edge set $\mathcal{E}$, the link interference matrix, $\mathbf{LIM}$, is an $E \times E$ symmetric matrix such that its $(a, b)^{th}$ ($a \neq b$) element is equal to 1 if (e_a, e_b) is a potentially interfering pair of edges.

$$\mathbf{LIM}_{ab} = \begin{cases} 1, & \text{if } e_b \in IE(e_a) \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

All diagonal elements of $\mathbf{LIM}$ are equal to 0 and row (column) a of the matrix $\mathbf{LIM}$ refers to the edge e_a.

It is interesting to note that the $\mathbf{LIM}$ matrix is essentially the adjacency matrix of the *interference graph*. Given a reachability graph $G = (\mathcal{N}, \mathcal{E})$ and the $\mathbf{LIM}$ matrix, the interference graph, $I(G)$, is a graph whose node set is the edge set of G and two nodes are connected by an edge in $I(G)$ if the corresponding elements in $\mathbf{LIM}$ are equal to 1. Specifically, the nodes e_a and e_b ($e_a, e_b \in \mathcal{E}$) in $I(G)$ are joined by an edge if $\mathbf{LIM}_{ab} = \mathbf{LIM}_{ba} = 1$.

Let $\mathbf{C} = [\mathbf{C}_{ef} : 1 \leq e \leq E, 1 \leq f \leq F]$ denote the channel assignment matrix such that $\mathbf{C}_{ef} = 1$ if edge e is assigned channel f and is equal to 0 otherwise. The collision domain of edge e_a, in quadratic form, is then given by:

$$\sum_f \mathbf{C}_{af} \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab} \mathbf{C}_{bf} \quad (4)$$

The primal formulation for *Problem P*-2 can therefore be written as shown in Figure 1. Note that the primal form

minimize $\sum_{e_a \in \mathcal{E}} \sum_f \mathbf{C}_{af} \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab} \mathbf{C}_{bf}$

subject to

$$\sum_f \mathbf{C}_{ef} = 1;\; \forall e \in \mathcal{E}$$

$$\mathbf{C}_{ef} \in \{0, 1\};\; \forall e \in \mathcal{E}, \forall f \in \mathcal{F}$$

Fig. 1. Primal quadratic model for *Problem P*-2.

involves a penalty minimization objective. If instead we attempt a reward minimization objective, we get the dual of the above model. Specifically, if edges e_a and e_b are potentially interfering, they each obtain an unit reward if they are assigned different frequencies. In the terminology of the MAX k-CUT (Definition 1), we refer to the edge between e_a and e_b in the interference graph representation of $\mathbf{LIM}$ as the *cut edge*. For edge e_a, the total reward is therefore:

$$\sum_f \mathbf{C}_{af} \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab} \left(1 - \mathbf{C}_{bf}\right) \tag{5}$$

Noting that the index e_b in the above expression can be changed to $e_b > e_a$ so that rewards are counted only once (not for e_a and e_b both), we have the dual of the optimization model in Figure 1, as shown in Figure 2.

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maximize $\sum_{e_a \in \mathcal{E}} \sum_f \mathbf{C}_{af} \sum_{e_b:\, e_b > e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab}(1 - \mathbf{C}_{bf})$
subject to

$$\sum_f \mathbf{C}_{ef} = 1;\ \forall e \in \mathcal{E}$$

$$\mathbf{C}_{ef} \in \{0, 1\};\ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}$$

Fig. 2. Dual quadratic model for *Problem P*-2.

This is exactly the formulation for the MAX k-CUT problem, where k is equal to F, the number of available channels. Edges which have been assigned the same channel will be referred to as belonging to the same cluster (or partition) in the context of the MAX k-CUT. While the optimal solutions for the primal and dual formulations in Figures 1 and 2 are the same, it has been shown by Sahni and Gonzalez [19] that finding an approximation algorithm for the primal version (which is analogous to the MIN k-PARTITION problem, Definition 2) is hard, but there exists a simple linear time factor $\left(1 - \frac{1}{k}\right)$ approximation algorithm for the dual version (MAX k-CUT). Relatively recent results on the hardness of the MIN k-PARTITION and the MAX k-CUT problems can be found in [20]. We also note that a slightly improved factor $\left(1 - \frac{1}{k}\right)\left(1 + \frac{1}{2\Delta + k - 1}\right)$ algorithm has been suggested by Halldórsson and Lau [21], where $\Delta = \max_a \sum_b \mathbf{LIM}_{ab}$. However, we do not consider their algorithm any further since the improvement over Sahni and Gonzalez's algorithm is minimal for high Δ.

Figure 3 provides a high level description of the algorithm suggested in [19], which has been slightly modified to account for channel diversity (1). The time complexity of the algorithm is $O(N + E + F)$. In the context of MAX k-CUT, our modification attempts to make the distribution of the nodes in the clusters as equitable as possible, without affecting the $\left(1 - \frac{1}{F}\right)$ approximation guarantee. Note that this approximation factor is for the dual version of *Problem P*-2 and does not translate in general to the primal version.

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1. **Given**: $\mathcal{E}$, $\mathbf{LIM}$ and F. Assume that $E > F$.
2. Let $SET(e)$ denote the cluster to which edge e is assigned ($SET(e) = 0$ if edge e has not yet been assigned a cluster).
3. Let $WT(f)$ denote the weight of cluster f. The weight of cluster f is equal to the number of intra-cluster edges in the induced line graph corresponding to cluster f. A pair of edges, e_a and e_b, assigned to cluster f contributes an unit cost to $WT(f)$ if $\mathbf{LIM}_{ab} = 1$.
4. Arbitrarily order all edges $e \in \mathcal{E}$.
5. Assign the first F edges from the list to the F clusters, one in each cluster.
6. For all other edges, initialize $SET(e) = 0$.
7. Set $WT(f) = 0$ for $f = 1, 2, \ldots F$;
8. Increment $e = F + 1$;
9. **while** ($e \leq E$)
   - Let $WT_{temp}(f)$ be the weight of cluster f with edge e included in cluster f. /* *Note that inclusion of edge e in cluster f may increase the weight of f by more than 1.* */
   - Find the cluster, say f^*, such that:
     $f^* = \text{argmin}_f \{WT_{temp}(f) : f = 1, 2, \ldots F\}$
   - If there is more than one cluster which satisfies the above condition, choose f^* such that the assignment is most channel diverse (see (1) and the subsequent discussion). Break ties arbitrarily, if required. /* *This step makes the algorithm channel diversity aware.* */
   - Assign $SET(e) = f^*$;
   - Assign $WT(f^*) = WT_{temp}(f^*)$;
   - Increment $e = e + 1$;

   end while
10. Output the channel assignments $\{SET(e) : e = 1, 2, \ldots E\}$ and the cost of the primal formulation $\sum_f WT(f)$.

Fig. 3. High level description of a channel diversity aware factor $\left(1 - \frac{1}{F}\right)$ approximation algorithm for the dual version of *Problem P*-2 (see Figure 2).

VI. Minimization of the Maximum Size of a Co-Channel Interference Set

The optimization models for *Problem P*-2 can be easily modified if minimization of the maximum size of a co-channel interference set is the objective. Denoting the maximum size of any co-channel interference set by t,

$$t = \max\left(\mathbf{C}_{af} \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab}\mathbf{C}_{bf};\ \forall e_a \in \mathcal{E}, \forall f \in \mathcal{F}\right) \tag{6}$$

the primal quadratic and linearized formulation for *Problem P*-3 can be written straightforwardly as shown in Figure 4.[4] Observe that *Problem P*-3 can be interpreted as a MIN-MAX version of the k-PARTITION problem, which is defined below:

Definition 5 (MIN-MAX k-PARTITION): Given a graph $G = (V, E)$ and a positive integer k, find a partition of V into k clusters such that the maximum of the node indegrees is minimized.

[4] While stronger linear formulations are certainly possible, the intent behind our formulation is simply to point out the structural similarities between *Problems P*-2 and *P*-3 and known problems in combinatorial optimization such as MAX k-CUT and MIN k-PARTITION.

To the best of our knowledge, no approximation algorithm has yet been proposed for the MIN-MAX k-PARTITION problem. However, the following existence result is known, due to Lovasz:

Theorem 1 ([22]): Let $G = (V, E)$ be a graph, $\Delta(G)$ the maximum node degree in G and let $t_1, t_2, \dots, t_k$ be k non-negative integers such that $t_1 + t_2 + \cdots + t_k \geq \Delta(G) - k + 1$. Then, V can be partitioned into k subsets $\{V_1, V_2, \dots, V_k\}$ inducing subgraphs $\{G_1, G_2, \dots, G_k\}$ such that $\Delta(G_i) \leq t_i$ for all $1 \leq i \leq k$.

It therefore immediately follows from Theorem 1 that:

Corollary 1: Let $G = (V, E)$ be a graph, $\Delta(G)$ the maximum node degree in G and let $t_1, t_2, \dots, t_k$ be k non-negative integers such that $t_1 + t_2 + \cdots + t_k \geq \Delta(G) - k + 1$. Then, V can be partitioned into k subsets $\{V_1, V_2, \dots, V_k\}$ inducing subgraphs $\{G_1, G_2, \dots, G_k\}$ such that

$$\max_i \ \Delta(G_i) \leq \left\lceil \frac{\Delta(G) - k + 1}{k} \right\rceil ; 1 \leq i \leq k$$

Noting that $\Delta(G) = \sum_b \mathbf{LIM}_{ab}$ and $k = F$, we have an upper bound for t (6).

$$t = \max_i \ \Delta(G_i) \leq \left\lceil \frac{\sum_b \mathbf{LIM}_{ab} - F + 1}{F} \right\rceil ; 1 \leq i \leq F \quad (7)$$

This bound can serve as an useful benchmark to compare the performance of heuristic algorithms since exact solution of the linearized ILP formulation in Figure 4 may be computationally intensive for dense graphs ($E \gg 1$).

Quadratic Formulation

minimize t

subject to

$$t - \mathbf{C}_{af} \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab} \mathbf{C}_{bf} \geq 0; \ \forall e_a \in \mathcal{E}, \forall f \in \mathcal{F}$$

$$\sum_f \mathbf{C}_{ef} = 1; \ \forall e \in \mathcal{E}$$

$$\mathbf{C}_{ef} \in \{0, 1\}; \ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}$$

Linearized Formulation

minimize t

subject to

$$t - \sum_{e_b:\, e_b \neq e_a,\, e_b \in \mathcal{E}} \mathbf{LIM}_{ab} \mathbf{Z}_{abf} \geq 0; \ \forall e_a \in \mathcal{E}, \forall f \in \mathcal{F}$$

$$\mathbf{C}_{af} + \mathbf{C}_{bf} - \mathbf{Z}_{abf} \leq 1;$$

$$\sum_f \mathbf{C}_{ef} = 1; \ \forall e \in \mathcal{E}$$

$$\mathbf{Z}_{abf} \in \{0, 1\}; \ \forall e_a, e_b \in \mathcal{E}, \forall f \in \mathcal{F}$$

$$\mathbf{C}_{ef} \in \{0, 1\}; \ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}$$

Fig. 4. Primal quadratic and linearized models for *Problem P*-3.

We now discuss a heuristic algorithm for *Problem P*-3. While, in general, minimizing the average collision domain size does not minimize the maximum collision domain size and *vice versa*, simulations suggest that the quality of the solutions for *Problem P*-2 is reasonably good when evaluated according to the criterion of *Problem P*-3. Our algorithm for *Problem P*-3 therefore consists of two phases; in the first phase, we run the algorithm in Figure 3, which is followed by a simple *local swap operation* to further reduce the maximum collision domain size. Intuitively, the swap operation involves checking if removing the edge e (or node e in the interference graph corresponding to $\mathbf{LIM}$) with the maximum indegree from its assigned cluster and assigning it to a different cluster reduces the objective function cost. If so, e is reassigned to the new cluster which results in a maximum reduction of the objective cost. This procedure is repeated until no further improvement is possible. Details of the composite algorithm are provided in Figure 5, which is self-explanatory. We note that, while the algorithm is primarily intended to reduce the maximum interference domain, it can also be used as an improvement heuristic for further reducing the average collision domain size. In this case, one can easily modify the algorithm so that a local swap is carried out only if there is a corresponding reduction in the average interference domain size. It can be shown that the worst case time complexity of the algorithm is $O\left(E^2 F\right)$.

In Figure 6, we show the channel assignments for *P*-2 and *P*-3 on a 6×6 grid, for $F = 4$. Observe that the improvement heuristic (Figure 5) has been able to simultaneously reduce both the average and maximum interference domain sizes in this case. This is however a coincidence and may not be generally true. Also, the procedure is not guaranteed to yield an improved solution (but the solution can be no worse than the original); this happens, for instance, if the algorithm is run for the 6×6 grid with $F = 3$ channels.

VII. Conclusion

We have considered the static channel assignment problem for multi-radio, multi-channel 802.11 wireless mesh networks. We presented four metrics based on which mesh channel assignments can be obtained. In particular, we have focussed on minimization of the average and maximum collision domain sizes and showed that these problems are closely related to problems in combinatorial optimization such as MAX k-CUT and MIN k-PARTITION. We have also presented heuristic algorithms for solving the channel assignment problems using the above two metrics. Currently, we are conducting system level simulations to compare the performance of the different channel assignment metrics, w.r.t end-to-end throughput and delay. These will be reported in a subsequent paper.

References

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1. **Given**: $\mathcal{E}$, **LIM** and F. Assume that $E > F$.

2. Run the algorithm in Figure 3. The channel assignments obtained are completely characterized by the $E \times 1$ array, SET, whose e^{th} element is given by $SET(e) = f$ if e is in cluster f.

3. Given a clustering, we refer to the node with the highest indegree as the *critical node*, denoted by cr_node: $cr\_node = \text{argmax}_i \delta_{i(f)}$, where $\delta_{i(f)}$ is the indegree of node i, assumed to be in cluster f. The indegree of the critical node is referred to as the *critical cost* and denoted by cr_cost.

2. Set $flag = 1$;

3. **while** $(flag)$

- Identify the critical node in the current clustering. Let cr_node be in cluster f, with corresponding cost cr_cost.
- Temporarily assign cr_node to all clusters other than f and recompute the corresponding node indegrees and critical costs.
- **if** (there is a reduction in critical cost due to the temporary reassignment)
  - Identify the cluster f^* such that reassigning cr_node to f^* results in the maximum reduction in critical cost
  - Assign $SET(cr\_node) = f^*$;

  else
  - Set $flag = 0$;

end while

10. Output the channel assignments $\{SET(e) : e = 1, 2, \ldots E\}$ and cr_cost.

Fig. 5. High level description of a heuristic algorithm for minimizing the maximum collision domain size (*Problem P*-3).

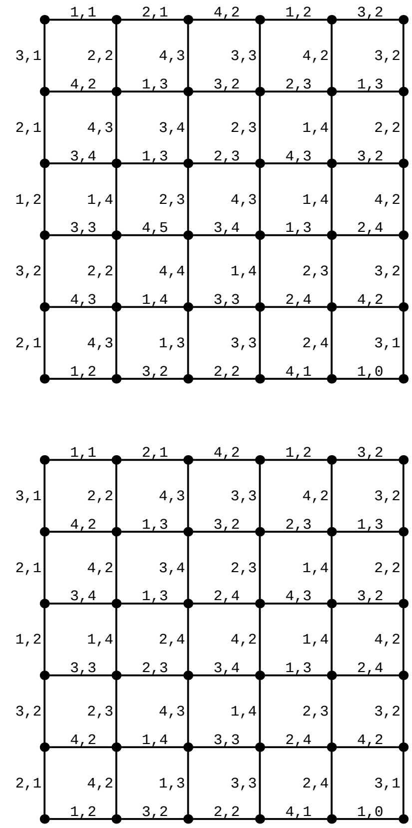

Fig. 6. (a) The top plot shows the channel assignments when the average size of a co-channel interference set is minimized for $F = 4$. The labels on the edges have 2 parameters. The first parameter represents the channel assigned to the edge and the second parameter represents the size of its co-channel interference set. The average (after scaling by $1/E$) interference domain size is 2.63 and the maximum interference domain size is 5. The channel diversity index is $MAXUSAGE - MINUSAGE = 16 - 14 = 2$.
(b) The bottom plot shows the channel assignments when the maximum size of a co-channel interference set is minimized for $F = 4$. The average interference domain size is 2.57 and the maximum interference domain size is 4. Observe that the improvement heuristic (Figure 5) has been able to simultaneously reduce both the costs in this case. The channel diversity index is $MAXUSAGE - MINUSAGE = 16 - 13 = 3$

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