Analysis of Throughput and Energy Efficiency of p-persistent CSMA with Imperfect Carrier Sensing

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***Abstract*— The drive towards portable wireless devices capable of forming ad-hoc wireless networks on demand has spurred significant interest in the design and analysis of power-efficient schemes at all layers of the protocol stack. In order to increase useful lifetimes of such battery-powered devices, it is vital to understand the role of carrier-sense based multiple access protocols in controlling the trade-offs between throughput and energy consumption performance. Our focus in this work is the *impact of imperfect carrier sensing* on this trade-off; accordingly, we present an analysis of the energy efficiency of p-persistent CSMA with carrier sense imperfections.**

I. Introduction and Literature Review

The success of wireless-LAN technologies like the IEEE 802.11 has lead to renewed interest in ad-hoc networks using low cost wireless devices that are deployed in large numbers. Wireless sensor networks are an example of such ad-hoc networks. Two typical characteristics of these low cost nodes are that they (a) have simple transceiver architectures and (b) operate on limited energy sources, both owing to the constraints on size and cost to exploit the economies of scale. On one hand, the simple transceiver architectures inherently consume less energy, while on the other they cause undesirable protocol imperfections which could potentially increase the energy consumption. There is thus an evident interplay between the transceiver design and MAC energy consumption.

CSMA is an important contention based medium access protocol used widely in wireless network systems. Specifically, p-persistent CSMA is the basis for several WLAN MAC protocols. In fact, a suitable p-persistent CSMA model [1] has been shown to well approximate the basic 802.11 MAC protocol [2] (Distributed Coordination Function or DCF) if p is so chosen as to ensure the same average backoff interval as the standard, i.e. $p = 1/(E[B] + 1)$.

The performance of CSMA protocols with *perfect* carrier-sensing was analyzed in depth in [3]. The effect of carrier-sense imperfections on the throughput of non-persistent and 1-persistent CSMA protocols was first studied in [4], [5] and p-persistent CSMA in [6]. The analytical model in the above references presumed infinite nodes with an aggregate Poisson arrival of packets. Energy efficiency analysis of p-persistent CSMA was first carried out in [7], wherein the system model consists of M active nodes with perfect carrier sensing, operating in asymptotic conditions and generating geometrically distributed packet lengths. To the authors' knowledge, the effect of imperfections on the energy consumption of p-persistent CSMA has not been analyzed in the past.

The analyses of CSMA schemes with imperfect carrier sensing in [4], [5], [6] and [7] are rather cumbersome. Here, we use a different (and in our view, readily generalizable) approach based on state transitions and flow graphs to determine the transfer functions between states, borrowing a technique widely employed in the analysis of code acquisition of spread-spectrum systems [8], [9]. We extend the application of signal flow graphs to the analysis of energy-efficiency and throughput of p-persistent CSMA and to characterize the impact of carrier sensing imperfections on its performance. We introduce a new metric the *throughput achieved per unit energy consumed*, η that simultaneously captures the effect of imperfections on energy consumption and throughput.

The paper is organized as follows. Section II describes our system model. In Section III, we provide a thorough analysis throughput and energy efficiency of p-persistent CSMA with and without carrier-sensing imperfections using the transfer function approach. In Section IV, simulation results are presented to substantiate the analysis and to verify the validity of the assumptions made. Concluding remarks are summarized in Section V.

II. System Description

We consider a system of M active nodes employing the p-persistent CSMA scheme [10] on a slotted shared channel, where each node senses the channel persistently in every time slot. If the node finds the channel to be idle, it transmits a packet of fixed length, N time slots with a probability $p = 1-q$ starting from the next time slot.[1] On the other hand, if the node finds the channel to be busy, it persists with sensing until the channel goes idle at which time it attempts a transmission again with probability p. In the event of a collision, it is assumed for the sake of simplicity that the collided packet gets rescheduled for transmission without additional time or energy cost, i.e. the acknowledgement is instantaneous. In order to derive closed-form formulas, we assume that the nodes operate in *asymptotic* (saturation) conditions, i.e. they always have a message waiting to be transmitted.

The carrier sensing operation performed by the nodes is seldom perfect. At the radio or link level, this is usually implemented using non-coherent energy detectors as a *quick and dirty* process with low overhead. Hence, each node has a

This work was supported in part by the National Science Foundation under Grant ANI 0325014 (ITR: RESCUENET-Embedded In-Building Sensor Network to Assist Disaster Rescue).

[1] It is assumed that the node uses a significant fraction of a slot duration for channel sensing, which explains why the transmission can begin only from the next slot.

non-zero probability of incorrectly sensing the nature of the channel. We characterize these imperfections as follows:

1) $P_{fa} = 1 - P'_{fa}$ is the probability that a node incorrectly senses an idle channel to be busy.
2) $P'_d = 1 - P_d$ is the probability that a node incorrectly senses a busy channel to be idle.

Our intention in this work is to characterize the impact of carrier-sense imperfections on the throughput of p-persistent CSMA and the associated energy consumption. Due to our assumption that the nodes are operating in saturation conditions, at any point they are only involved in either sensing the channel or in transmission (there is no reception). Therefore, from an energy consumption standpoint, each node is assumed to alternate between two states:

1) a carrier-sense state, with an energy consumption of E_{cs} per time slot and
2) a transmit state, with E_t per time slot.

III. ENERGY EFFICIENCY AND THROUGHPUT ANALYSIS

Throughput of a node is defined as the fraction of time slots where messages are successfully transmitted. **Energy efficiency** is the fraction of the total energy spent by a node used for the above successful transmissions. As explained in [7] (and as we will see shortly), throughput analysis can be carried out as a special case of the energy efficiency analysis. Hence, in this paper we concentrate only on the latter and customize the results to determine the throughput as needed.

Considering the p-persistent protocol behavior, all the processes that characterize the energy consumption of a node are regenerative when a successful transmission occurs from the node. Moreover, when the channel is idle, the processes are memoryless, i.e. the node's behavior at the next time slot is independent of the past. With this observation, we identify the states of the system as described next and compute the transfer function from the point when the channel is idle till the end of one successful transmission (the regenerative interval). Finally, an expression for the mean energy spent by one node (the *tagged* node), to transmit one packet successfully is computed.

The analysis is carried out in two stages. First, we perform an accurate analysis of energy efficiency with all the nodes in the system capable of doing perfect carrier sensing in III-A. Following this, an approximate analysis of energy efficiency is presented in III-B with carrier sensing being imperfect.

A. Perfect carrier sensing

Let us define the following to be the states of the system.

1) **TX**: The tagged node has captured the channel and is successfully transmitting.
2) **COL**: The tagged node is involved in a collision, i.e. a different node has started transmission at the same time as the tagged node.
3) **IDLE**: The channel is idle i.e. no node is transmitting
4) **BUSY**: The channel is busy i.e. some node other than the tagged one has captured the channel and is transmitting.

Although the process with the states as described above is not of Markovian nature, the sequence of events corresponding to the state transitions forms an embedded Markov chain. With this inference, we could go on to determine the probability generating function of the n-step transition probabilities, $p_{ij}(n)$ from state i to state j as in [11], i.e.

$$P_{ij}(z) \triangleq \sum_{n=0}^{\infty} p_{ij}(n) z^n \tag{1}$$

It turns out that $P_{ij}(z)$ represents the transfer function of

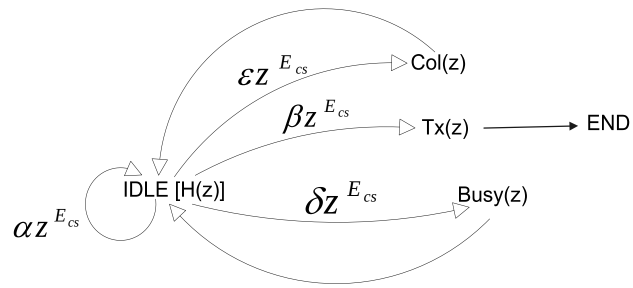

Fig. 1. State diagram description of p-persistent CSMA with no imperfections

an equivalent signal flow graph obtained by assigning a gain of $p_{ij}z$ to each transition from i to j of the state transition diagram, where p_{ij} is the one step transition probability and z is the unit energy operator [12]. From the equivalent flow graph, the transfer function can be derived rather easily using flow graph reduction methods. Accordingly, the flow graph for the above defined states for the case of all nodes being perfect is given in Fig. 1. In the graph, Tx(z), Busy(z) and Col(z) are the transfer functions from entering states **TX**, **BUSY** and **COL** respectively to exiting them.[2] If we define $H(z)$ as the transfer function starting from the **IDLE** state till the end of a successful transmission, then from the flow graph

$$\begin{aligned} H(z) &= z^{E_{cs}} \left[\alpha H(z) + \beta \text{Tx}(z) + \delta \text{Busy}(z) H(z) \right. \\ & \quad \left. + \epsilon \text{Col}(z) H(z) \right] \\ &= \frac{\beta z^{E_{cs}} \text{Tx}(z)}{1 - z^{E_{cs}} \left[\alpha + \delta \text{Busy}(z) + \epsilon \text{Col}(z) \right]} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \alpha &= q^M, & \beta &= pq^{M-1}, \\ \delta &= q(1 - q^{M-1}), & \epsilon &= p(1 - q^{M-1}) \end{aligned} \tag{3}$$

Once the node has entered the **TX** state, it exits the system after spending energy NE_t indicating a successful packet transmission. From **COL** state the node returns to **IDLE** state after an expenditure of energy NE_t since the transmission is unsuccessful. When in **BUSY** state the node has to sense the channel for N time slots and thus spend NE_{cs} energy at the end of which it has to return to **IDLE** state. Therefore,

$$\text{Tx}(z) = z^{NE_t}; \quad \text{Busy}(z) = z^{NE_{cs}}; \quad \text{Col}(z) = z^{NE_t} \tag{4}$$

Now, the total mean energy spent for one successful transmission can be determined by

[2] The information about the energy/time spent in each state will be subsumed in its respective transfer function.

$$E = \frac{d}{dz}H(z)\Big|_{z=1} \tag{5}$$

which after some algebra can be shown to be

$$E = \left(\frac{1+N\delta}{\beta}\right)E_{cs} + \left(\frac{N\beta+N\epsilon}{\beta}\right)E_t \tag{6}$$

The mean time spent between two successful transmissions, T can then obtained by simply viewing z as the unit delay operator and using $E_t = 1$ and $E_{cs} = 1$ in (6). The energy efficiency and throughput of the tagged node are then given by [3]

$$\rho = \frac{N}{T}; \qquad \rho_{energy} = \frac{NE_t}{E} \tag{7}$$

B. Imperfect carrier sensing

A full analysis of throughput and energy efficiency allowing imperfections at all nodes is cumbersome and may not provide good insight into the impact of such imperfections on aggregate network metrics. We therefore make some model assumptions to simplify analysis without relinquishing the essence. We define first the following states of the system.

1) **IDLE**: The channel is idle and all nodes are waiting to transmit.
2) **TX**$_{succ}$: The tagged node has successfully captured the channel and its transmission is successful, i.e. no other node interferes with its transmission.
3) **TX**$_{fail}$: The tagged node has successfully captured the channel but its transmission is unsuccessful because a different node begins transmission due to its imperfection causing a *partial* collision.
4) **BUSY**: The channel has been captured by a node other than the tagged node.
5) **COL**: The tagged node and another node have begun transmission at the same time and are thus involved in a *full* collision.

For the sake of conciseness, we define two additional probabilities. $p_i = 1 - q_i = P'_{fa}p$ is the probability that a node transmits when the channel is idle and $p_b = 1 - q_b = P'_d p$ is the probability that a node transmits when the channel is busy.

The flow graph with the states defined as above is shown in Fig. 2. The transfer function starting from the **IDLE** till the end of a successful transmission is then given by

$$\begin{aligned} H(z) = z^{E_{cs}} [&\alpha H(z) + \beta \mathrm{Tx}_{succ}(z) + \beta \mathrm{Tx}_{fail}(z)H(z) \\ &+ \delta \mathrm{Busy}(z)H(z) + \epsilon \mathrm{Col}(z)H(z)] \end{aligned}$$

$$= \frac{\beta z^{E_{cs}} \mathrm{Tx}_{succ}(z)}{1 - z^{E_{cs}}[\alpha + \beta \mathrm{Tx}_{fail}(z) + \delta \mathrm{Busy}(z) + \epsilon \mathrm{Col}(z)]} \tag{8}$$

where

$$\begin{aligned} \alpha &= q_i^M, & \beta &= p_i q_i^{M-1}, \\ \delta &= q_i(1-q_i^{M-1}), & \epsilon &= p_i(1-q_i^{M-1}) \end{aligned} \tag{9}$$

[3]Due to the symmetry of the system, the aggregate throughput of the system is just $M\rho$

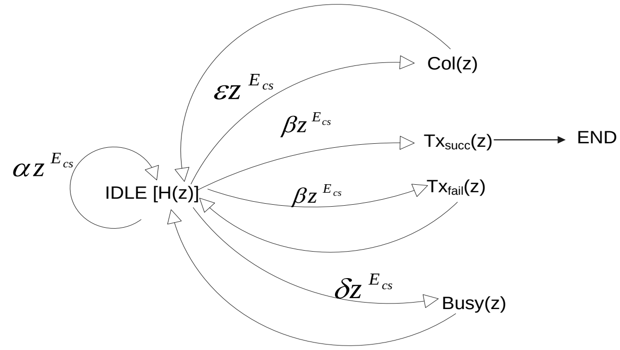

Fig. 2. State diagram description of p-persistent CSMA with all nodes being imperfect

For the transmission from the tagged node to be successful, the other $(M-1)$ nodes should refrain from transmitting at all the N time slots of its transmission, in which case the tagged node exits the system with an expense of energy NE_t.

$$\mathrm{Tx}_{succ}(z) = (q_b^{M-1})^N z^{NE_t} \tag{10}$$

If transmission from the tagged node is interrupted by incorrect sensing of transmission from another node, its NE_t energy spent is wasted and the channel remains busy for a few additional slots due to the presumed transmission. Although there is a finite probability that the channel remains busy for more than N slots due to a series of partial collisions or that the tagged node could begin another transmission before the channel goes idle, we assume these to be negligible. Partial collisions are simply an effect of P_d; for high P_d, the above is a reasonable assumption.

$$\mathrm{Tx}_{fail}(z) = z^{NE_t}\sum_{n=0}^{N-1}(q_b^{M-1})^n(1-q_b^{M-1})z^{(n+1)E_{cs}} \tag{11}$$

When the channel is busy, the tagged node spends only NE_{cs} units of energy before returning to the **IDLE** state if all the $(M-1)$ nodes refrain from transmitting at the N time slots of the current transmission. If one of the nodes (other than the tagged node) causes a partial collision, the channel busy period gets extended by a few additional slots. As a third possibility, if the tagged node begins transmission as a result of an incorrect sensing, it incurs a NE_t energy loss due to a wasteful transmission. Again, although a sequence of more than two collisions is likely to happen, we assume that the probability of such an event is negligible. Therefore,

$$\begin{aligned} \mathrm{Busy}(z) = z^{NE_{cs}}(q_b^{M-1})^N + z^{NE_t}\sum_{n=0}^{N-1}q_b^n(1-q_b)z^{(n+1)E_{cs}} \\ + z^{NE_{cs}}q_b^N\sum_{n=0}^{N-1}(q_b^{M-2})^n(1-q_b^{M-2})z^{(n+1)E_{cs}} \end{aligned} \tag{12}$$

We assume that if a full collision happens, it only involves two nodes, i.e the probability of more than two nodes beginning transmission at the same slot is negligible. This assumption is reasonable for small p values, which is anyway appropriate for large M for reasonable throughput. In the event of a full collision, the tagged node suffers a wasteful energy loss of NE_t if the other $(M-2)$ nodes refrain from transmitting for N consecutive time slots. If one of them starts an incorrect transmission, the channel is kept busy for a few additional slots thus incurring extra energy spent in carrier sensing. Therefore,

$$\begin{aligned}\mathrm{Col}(z) &= z^{NE_t}(q_b^{M-2})^N \\ &\quad + z^{NE_t}\sum_{n=0}^{N-1}(q_b^{M-2})^n(1-q_b^{M-2})z^{(n+1)E_{cs}}\end{aligned} \tag{13}$$

The mean energy expended by the tagged node in transmitting one packet successfully can be determined from (5). After some algebra, it can be shown to be

$$E = \left(\frac{N_{cs}}{\beta q_b^{N(M-1)}}\right)E_{cs} + \left(\frac{N_t}{\beta q_b^{N(M-1)}}\right)E_t \tag{14}$$

where

$$\begin{aligned}N_t &= N\left(1-\alpha-\delta q_b^N\right) \\ N_{cs} &= 1+\beta\left(\frac{1-q_b^{N(M-1)}}{1-q_b^{M-1}} - Nq_b^{N(M-1)}\right) \\ &\quad +\delta\left(q_b^N\frac{1-q_b^{N(M-2)}}{1-q_b^{M-2}} + \frac{1-q_b^N}{1-q_b} - Nq_b^{N(M-1)}\right) \\ &\quad +\epsilon\left(\frac{1-q_b^{N(M-2)}}{1-q_b^{M-2}} - Nq_b^{N(M-2)}\right)\end{aligned} \tag{15}$$

When $P_d = 1$ and $P_{fa} = 0$ (14) reduces to (6). Again the mean time spent in one successful transmission can be obtained by plugging $E_{cs} = 1$ and $E_t = 1$ in (14).

IV. Simulations

In this section, we verify the analysis and validate the assumptions made by means of simulations. All the simulations have been carried out in Matlab. Fig. 3 shows the energy efficiency as a function of parameter p with perfect and imperfect carrier sensing for $M = 10$ and $M = 20$. For these simulations a $(E_{cs} : E_t)$ ratio of $(1 : 5)$ and $N = 10$ have been used. The first observation from the figure concerns the validity of the assumptions made in the analysis. The analysis produces very accurate results for small values of p. At large p values, there are too many collisions in the system and our assumptions regarding a series of two or more partial collisions and that of more than two nodes being involved in a full collision being small do not hold. It is also evident from the figure that at reasonable values of p, the results predicted by the analysis matches the simulations very well for values of P_d and P_{fa} up to 0.9 and 0.1 respectively. Since typical $P_d - P_{fa}$ values are expected to be much better than these, the analysis should be very useful in predicting accurate results.

For every choice of M and N, the performance is characterized by the presence of an optimum p. It is seen from Fig. 3 that this optimum p decreases as the imperfections in the system increase. In [7], expressions for the optimum p have been derived for the case of no imperfect nodes. When

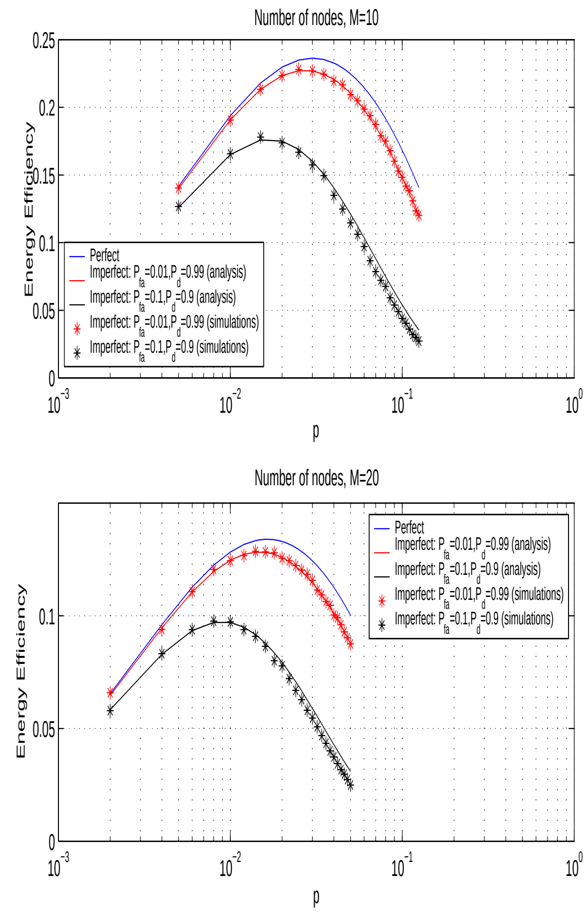

Fig. 3. Energy Efficiency against parameter p

imperfections are anticipated in the system, the value of p for optimum performance has be to be chosen to be less than that for the case of all perfect nodes and can be computed numerically from (14).

Fig. 4 shows the variation of energy efficiency with the number of nodes, M. For each M, the value of p has been chosen to be $0.5/M$ and $N = 10$. It is clear from this figure that the effect of the simplifying assumptions made in the analysis is relatively insensitive to M and the analysis results match the simulations very well as long as p is so chosen as to avoid too many collisions.

In Fig. 5 the energy efficiency is plotted as a function of the packet length, N for $M = 10$ and $p = 0.05$. An interesting observation from this figure is that while the energy efficiency performance keeps improving with N when all nodes in the system are perfect, the case of imperfect carrier sensing has an optimum N value at which the energy efficiency is maximum. This is because while large packet sizes reduce the fraction of energy spent on overheads, they also increase the chances of collisions due to imperfect sensing by the other nodes. Also, more imperfections in the system call for smaller packet size for increased efficiency. It is also clear from the figure that the accuracy of our analysis degrades with increasing packet lengths. This is simply because of the fact that the chance of partial collisions increases with increasing packet lengths,

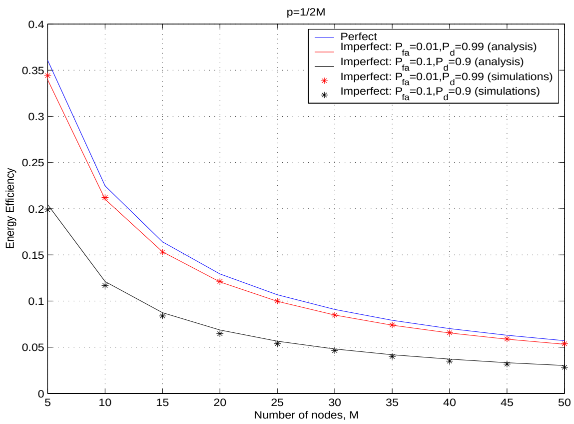

Fig. 4. Energy Efficiency against number of nodes, M

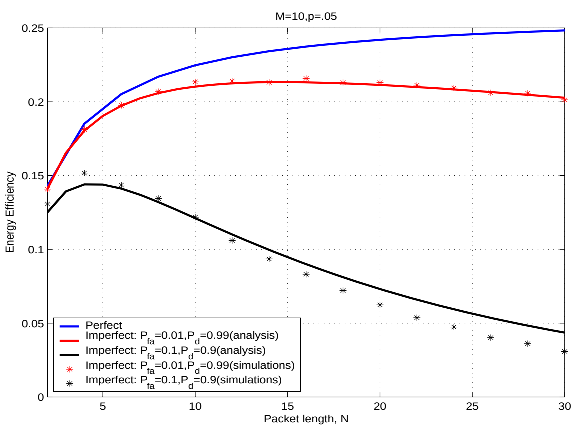

Fig. 5. Energy Efficiency against packet size, N

which invalidates our assumption that the probability of a sequence of more than two partial collisions is negligible.

Since carrier sense imperfections cause the throughput to drop and the energy consumption to raise, it would be useful to quantify the combined effect of imperfections on throughput and energy efficiency. To this end, we introduce a new metric, the *throughput achieved per unit energy consumed*, η. η is a more relevant metric as a basis to compare various energy saving protocol modifications. It is plotted in Fig. 6 as a function of the parameter p for $M = 10$ and $N = 10$. The effect of carrier sense imperfections is much clearer in this figure: as the imperfections increase, η drops significantly. Carrier sense imperfections cause energy and time to be wasted both in idle listening (low p values) and in collisions (high p values) and when coupled with a poor choice of parameter p could result in unacceptably low performance as is evident from the figure. This underscores the need to choose p judiciously by taking the carrier sense imperfections in to consideration.

V. Conclusions

An accurate analysis of the energy efficiency of a system employing p-persistent CSMA has been presented using the transfer function approach for the case of all nodes in the system being perfect in their carrier sensing capabilities. For the

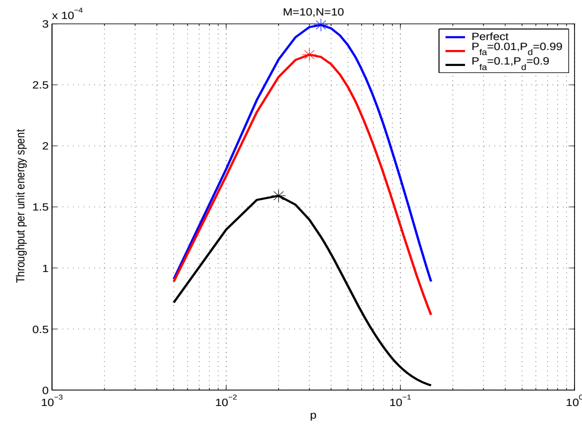

Fig. 6. Throughput achieved per unit energy consumed, η vs p

case of imperfect carrier sensing, an approximate analysis with reasonable simplifying assumptions has been presented. The analysis has been shown to provide very accurate results when the number of nodes is large (which implies that p is small) and P_d and P'_{fa} values are high. While simple transceiver architectures inherently consume less energy, they are also associated with undesirable carrier sensing imperfections that could potentially increase energy consumption indicating a trade-off between complexity and energy consumption. If the statistics of imperfections and energy consumption characteristics of transceivers of varying complexities are known, the analysis presented here would prove useful in selecting the right complexity for optimum energy efficiency.

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