A Rate-One Non-Orthogonal Space-Time Coded OFDM System with Estimation for Frequency Selective Channels*

Lei Shao and Sumit Roy**
Dept. of Electrical Engineering, University of Washington, BOX 352500, Seattle, WA 98195
Email: {ls7, roy}@ee.washington.edu

Abstract In this paper, we investigate the impact of applying *the channel estimation method in [1] to an OFDM system* equipped with rate 1 non-orthogonal space time block code[2] for K transmit and 1 receive antenna. A new training pattern based on the Hadamard construction is designed that enables estimation of the K channels separately and thus reduces the effective noise variance in the estimate. The simulation result shows that using this low rank channel estimation in frequency selective slow fading channel yields very good BER performance comparable to the case when channel is known at the receiver. Moreover, the use of rate 1 *non-orthogonal* STBC achieves significantly superior performance vis-à-vis with Alamouti's *orthogonal* STBC.*

I Introduction

OFDM is an attractive candidate for next generation broadband wireless communication services, since dividing the available spectrum into many narrow parallel sub-channels allows efficient suppression of ISI in high symbol rate designs. Nonetheless, subcarriers that encounter a fading null (due to channel dispersion at these frequencies) will encounter high error probability, necessitating some form of error correction coding and diversity techniques as compensation. One way of achieving such diversity is by use of multiple transmit and receive antennas, leading to multiple input, multiple output (MIMO) systems. Space-time coding has been developed for high data rate MIMO wireless systems [3,4] for single carrier modulation and extended to OFDM in [5]. However, space-time codes developed to date assume perfect knowledge of the MIMO channel at the receiver. In spite of this, several good channel estimation methods have been proposed for space-time coded OFDM systems [6,7,8]. In [6], a Least Square (LS)–based channel estimation method was presented that jointly estimated the impulse responses of all channels based on an optimal training signal design. However, training sequences consume bandwidth and thus incur spectral efficiency losses especially in rapidly varying environments. For this reason, blind channel estimation methods have received much attention. In [8], a deterministic constant modulus (CM) blind channel estimator was proposed which can identify the channels under certain conditions. In [7], a semi-blind channel estimation method was proposed using a novel precoder that guaranteed channel identifiability regardless of channel zero location. The training based method was used to obtain initial channel estimates in [7] and the semi-blind method was shown to track slow channel variations.

On the other hand, it is well known that for space-time block codes, a rate one complex orthogonal design exists only for the two transmit antenna case. For more than two transmit antennas, generalized complex designs lead to sub-unity code rates. In [2], the construction of a rate one *non-orthogonal* class of space time (ST) blocks for more than two antennas was discussed under flat fading channels. The class of ST block codes proposed in [2] exists for an arbitrary number of transmit antennas. Although this class of ST block codes cannot achieve full diversity gain, the resulting gain is still considerable when sufficient number of transmit antennas are used.

In this paper, we consider a rate one non-orthogonal space-time coded OFDM system with application in fixed wireless or wireless LAN. The key contribution in our work is design of a new training pattern based on the Hadamard construction which enables accurate estimation of each individual channel using the low rank channel estimator and reduces the effective noise variance in the estimate. Moreover, it is verified by simulation that the rate 1 non-orthogonal STBC achieves significant gain relative to Alamouti's STBC.

II.A System Model

Fig.1 shows the rate-1 Non-Orthogonal S-T coded OFDM baseband system used in this work. Cyclic prefix (CP) is used to preserve the orthogonality of the tones and eliminate intersymbol interference (ISI) between consecutive OFDM symbols. Moreover, the channel is assumed to be slowly fading and is considered constant during four successive OFDM symbols. The number of tones in thes ystem is N and the length of the CP is L samples.

The information symbols are grouped into blocks $S(n)$ of dimension $N \times 1$ that are fed into the rate-1 non-orthogonal space time encoder. The ST encoder takes as input four consecutive blocks, $S(4n), S(4n+1), S(4n+2), S(4n+3)$ and outputs the following $4N \times 4$ code matrix:

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* This research is supported by the NSF ITR under the Grant No.624504.

** Author for all correspondence; phone: +1-206-221-5261; fax: +1-206-543-3842.

$$\begin{bmatrix} S(4n) & -S^*(4n+1) & S(4n+2) & -S^*(4n+3) \\ S(4n+1) & S^*(4n) & S(4n+3) & S^*(4n+2) \\ S(4n+2) & -S^*(4n+3) & S(4n) & -S^*(4n+1) \\ S(4n+3) & S^*(4n+2) & S(4n+1) & S^*(4n) \end{bmatrix} \quad (1)$$

where each column is transmitted in successive time intervals using the four transmit antennas, respectively. Note that without blocking $(N=1)$, the code matrix in Equation (1) is the same as the code matrix in (25) of [2].

For convenience, we denote by $S_i(n), i=1,2,3,4$, the $N\times 1$ block transmitted through the i th transmit antenna at the time n. Then, $S_i(n)$ is related to $S(n)$ as follows:

Column 1 of (1) corresponds to $S_i(4n), i=1,2,3,4$
Column 2 of (1) corresponds to $S_i(4n+1), i=1,2,3,4$
Column 3 of (1) corresponds to $S_i(4n+2), i=1,2,3,4$
Column 4 of (1) corresponds to $S_i(4n+3), i=1,2,3,4$ (2)

The attenuations on each tone are given by

$$h_{ij}=G_i\left(\frac{j}{NT_s}\right),\ j=0,\cdots,N-1, i=1,2,3,4 \quad (3)$$

where $G_i(\bullet)$ is the frequency response of the ith channel $g_i(\tau)$. Define $N\times 1$ block $H_i=[h_{i0}\ h_{i2}\cdots h_{i,N-1}]$, and $D_i=diag(H_i), i=1,2,3,4$. In matrix notation, we can describe the OFDM system as

$$\begin{aligned} r(n) &= \sum_{i=1}^{4} diag(S_i(n))H_i + w(n) \\ &= \sum_{i=1}^{4} D_i S_i(n) + w(n) \end{aligned} \quad (4)$$

where $r(n)$ and $w(n)$ denote the $N\times 1$ ISI-free received symbol block and additive white Gaussian noise vector after CP removal and FFT, respectively. Substituting (2) into (4), we can rewrite the above as

$$\begin{bmatrix} r(4n) \\ r^*(4n+1) \\ r(4n+2) \\ r^*(4n+3) \end{bmatrix} = \underbrace{\begin{bmatrix} D_1 & D_2 & D_3 & D_4 \\ D_2^* & -D_1^* & D_4^* & -D_3^* \\ D_3 & D_4 & D_1 & D_2 \\ D_4^* & -D_3^* & D_2^* & -D_1^* \end{bmatrix}}_{D} \begin{bmatrix} S(4n) \\ S(4n+1) \\ S(4n+2) \\ S(4n+3) \end{bmatrix} + \begin{bmatrix} w(4n) \\ w^*(4n+1) \\ w(4n+2) \\ w^*(4n+3) \end{bmatrix} \quad (5)$$

The non-orthogonal space-time decoder uses matrix D^H to decode the received signal matrix, yielding

$$z = \underbrace{\begin{bmatrix} A & \Phi & B & \Phi \\ \Phi & A & \Phi & B \\ B & \Phi & A & \Phi \\ \Phi & B & \Phi & B \end{bmatrix}}_{M} \begin{bmatrix} S(4n) \\ S(4n+1) \\ S(4n+2) \\ S(4n+3) \end{bmatrix} + D^H \begin{bmatrix} w(4n) \\ w^*(4n+1) \\ w(4n+2) \\ w^*(4n+3) \end{bmatrix} \quad (6)$$

where

$A=D_1^H D_1 + D_2^H D_2 + D_3^H D_3 + D_4^H D_4$,
$B=D_1^H D_3 + D_2^H D_4 + D_3^H D_1 + D_4^H D_2$

and Φ denotes a $N\times N$ zero matrix. Due to non-orthogonality, the matrix M is not diagonal, therefore $S(4n), S(4n+1), S(4n+2), S(4n+3)$ cannot be detected separately. Thus, a decorrelator denoted by matrix $M^{(1)}$ is used to generate

$$\begin{aligned} M^{(1)}z &= \begin{bmatrix} A & \Phi & -B & \Phi \\ \Phi & A & \Phi & -B \\ -B & \Phi & A & \Phi \\ \Phi & -B & \Phi & A \end{bmatrix} z \\ &= (A^2-B^2)\otimes I_4 \begin{bmatrix} S(4n) \\ S(4n+1) \\ S(4n+2) \\ S(4n+3) \end{bmatrix} + M^{(1)}D^H \begin{bmatrix} w(4n) \\ w^*(4n+1) \\ w(4n+2) \\ w^*(4n+3) \end{bmatrix} \end{aligned} \quad (7)$$

where $\otimes$ is the Kronecker product. The four outputs $\hat{S}^1(4n), \hat{S}^1(4n+1), \hat{S}^1(4n+2), \hat{S}^1(4n+3)$ denote the preliminary estimates; asi n [2], one stage of interference cancellation is used to generate the final decisions

$$\begin{bmatrix} \hat{S}^2(4n) \\ \hat{S}^2(4n+1) \\ \hat{S}^2(4n+2) \\ \hat{S}^2(4n+3) \end{bmatrix} = dec\left(z-(M-A\otimes I_4)\begin{bmatrix} \hat{S}^1(4n) \\ \hat{S}^1(4n+1) \\ \hat{S}^1(4n+2) \\ \hat{S}^1(4n+3) \end{bmatrix}\right) \quad (8)$$

II B. Channel Model

As in [1], we consider a fading multipath channel model, consisting of M impulses

$$g(\tau)=\sum_{k=0}^{M-1}\alpha_k\delta(\tau-\tau_k T_s) \quad (9)$$

where α_k are zero-mean complex Gaussian random variables with a power-delay profile $\theta(\tau_k)$. M=5 was used with an exponentially decaying power-delay profile $\theta(\tau_k)=Ce^{-\tau_k/\tau_{rms}}$ and delays τ_k that are uniformly and independently distributed over the length of the CP.

II C. Rate-1 Non-Orthogonal Space Time Block Codes

Complex, rate -1, orthogonal S-T block codes exist only for two transmit antennas; however, non-orthogonal rate-1 S-T block codes exist for arbitrary number of transmit antennas [2], while allowing a controlled number of interference terms per detected symbol.

As is well known for two transmit antennas, the Alamouti scheme [4] maps a pair of symbols $z = [z_1\ z_2]^T$ into the matrix $Z_{12} = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix}$, called an Alamouti block.

The corresponding received signal vector $r = [r_1\ r_2]^T$ is given by

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = Z_{12}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = Z_{12}h + n \qquad (10)$$

The receive equation can be expressed, equivalently, in terms of a channel matrix H_{12} via

$$\tilde{r} = \begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = H_{12}z + \tilde{n} \qquad (11)$$

In the case of non-orthogonal space time block codes, the conditions under which r and $\tilde{r}$ are interchangeable were discussed in [2]. The H matrix representation was preferred so that the design problem of constructing good codes was translated into the problem of designing spreading codes that reduce interference between data symbols. The solution to the latter problem is closely related to the class of linear real Hadamard codes. The case K=N=2p was first discussed in [2], where K is number of transmit antennas and N is the number of consecutive complex symbols to be encoded. Consider the class of S-T block codes for which the channel matrix H has the general form

$$H = \sum_{i=1}^{p}(C_i \otimes H_{2i-1,2i} + D_i \otimes H_{2i-1,2i}^*) \qquad (12)$$

where $H_{2i-1,2i}$ is an Alamouti block and C_i and D_i are $p \times p$ complex matrices. It is proved in [2] that

a) If $D_i = 0$ for $i = 1, \cdots, p$,

and $X_i, i = 1,2,\cdots, p/2$, form a p/2-canonical set (see Appendix for definition of p-canonical set), then a p-canonical set can be constructed as

$$C_{2i-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes X_i,\quad C_{2i} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes X_i, \forall i \qquad (13)$$

b) If both $C_i \neq 0$ and $D_i \neq 0$, for $i = 1, \cdots, p$, then a p-canonical set can be constructed as

$$C_{2i-1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes X_i,\quad C_{2i} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes X_i$$
$$D_{2i-1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \otimes X_i^H,\quad D_{2i} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \otimes X_i^H, \forall i \qquad (14)$$

If the number of transmit antennas K is arbitrary (not a power of 2), then m satisfies $2^{m-1} < K \le 2^m$, $(m \ge 2)$. A code Z is constructed for 2^m antennas, but antennas K+1 .... 2^m are not used for transmission, and the decoder sets $h_{K+1} = \cdots = h_{2^m} = 0$ in the H matrix.

After matched filtering at the receiver, we get.

$$y = \underbrace{H^H H}_{M} z + H^H \tilde{n} \qquad (15)$$

For non-orthogonal S-T block codes, M is not a diagonal matrix, so matrix $M^{(1)}$ is needed to decorrelate the signal y before detection.

Example: K=4

The following pair of matrices is a 2-canonical set

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (16)$$

which implies

$$H = \begin{bmatrix} H_1 & H_2 \\ H_2 & H_1 \end{bmatrix},\ Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_2 & Z_1 \end{bmatrix} \qquad (17)$$

where $H_i, i = 1,2$ is an Alamouti block.

III A. Simplified Low Rank Channel Estimation

When QPSK modulation is used, Equation (3) in [1] can be simplified into

$$\hat{H}_{lmmse} = R_{HH}(R_{HH} + \frac{1}{SNR}I)^{-1}\hat{H}_{LS} \qquad (18)$$

where

$$\hat{H}_{LS} = (diag(S))^{-1}y \qquad (19)$$

The optimal rank-p estimator can also be simplified to

$$\hat{H}_p = U\Delta_p U^H \hat{H}_{LS} \qquad (20)$$

where Δ_p is a diagonal matrix with entries

$$\delta_i = \begin{cases} \dfrac{\lambda_i}{\lambda_i + \frac{1}{SNR}}, & i = 1,2,\cdots p \\ 0, & i = p+1,\cdots,N \end{cases} \qquad (21)$$

The above equations are used to estimate each channel separately with the change that the $\frac{1}{SNR}$ term is replaced with $K\sigma_w^2 / 2^{m+1}$.

III B. Training block design

For a fair comparison between S-T coding methods with different number of transmit antennas, we fix the total power per bit at the receiver to be 1, so the power of each bit at a transmit antenna is 1/K, where K is the number of transmit antennas. Since QPSK is employed, the symbol power is 2/K on each transmit antenna. For simplicity, we first consider the K= 4 transmit antenna case. Four training blocks are used to estimate the four channels. Using Hadamard matrix as the training blocks, we can separately estimate the individual channels with simple linear combination of the received blocks.

Using a K=4 Hadamard matrix, we have the following:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \Rightarrow \begin{bmatrix} S_1 & S_2 & S_3 & S_4 \\ S_1 & -S_2 & S_3 & -S_4 \\ S_1 & S_2 & -S_3 & -S_4 \\ S_1 & -S_2 & -S_3 & S_4 \end{bmatrix} \quad (22)$$

where each row corresponds to one time instant, and each column corresponds to one transmit antenna. For OFDM system, S_1, S_2, S_3 and S_4 represent the 4 training blocks (4 OFDM symbols). Thus from (4)

$$\begin{aligned} r_1 &= D_1S_1 + D_2S_2 + D_3S_3 + D_4S_4 + w_1 \\ r_2 &= D_1S_1 - D_2S_2 + D_3S_3 - D_4S_4 + w_2 \\ r_3 &= D_1S_1 + D_2S_2 - D_3S_3 - D_4S_4 + w_3 \\ r_4 &= D_1S_1 - D_2S_2 - D_3S_3 + D_4S_4 + w_4 \end{aligned} \quad (23)$$

Therefore,

$$\begin{aligned} y_1 &\triangleq \frac{r_1 + r_2 + r_3 + r_4}{4} = D_1S_1 + \frac{w_1 + w_2 + w_3 + w_4}{4} \\ y_2 &\triangleq \frac{r_1 - r_2 + r_3 - r_4}{4} = D_2S_2 + \frac{w_1 - w_2 + w_3 - w_4}{4} \\ y_3 &\triangleq \frac{r_1 + r_2 - r_3 - r_4}{4} = D_3S_3 + \frac{w_1 + w_2 - w_3 - w_4}{4} \\ y_4 &\triangleq \frac{r_1 - r_2 - r_3 + r_4}{4} = D_4S_4 + \frac{w_1 - w_2 - w_3 + w_4}{4} \end{aligned} \quad (24)$$

The noise variance in the equations above has been reduced to $\frac{\sigma_w^2}{4}$. Also notice that $D_iS_i = diag(S_i)H_i$, thus we plug y_i and $S_i, i = 1,2,3,4$ into equations (19) and (20) to get optimal rank p estimate of each channel. If 3 transmit antennas are used, we remove the last column of the matrix above and still use four time instants to transmit the 3 training blocks. Correspondingly at the receiver, we only need to calculate y_1, y_2 and y_3 which identical to that in the K=4 case. In this way, when 3 transmit antennas are used, the new noise variance can still be reduced to $\frac{\sigma_w^2}{4}$. In conclusion, since the rate 1 non-orthogonal STBC can be applied to any number of antennas, if $2^{m-1} < K \le 2^m (m \ge 2$, m is an integer) transmit antennas are used, the method above (taking advantage of the $2^m \times 2^m$ Hadamard matrix, truncated if necessary), allows us to reduce the noise variance to $\frac{\sigma_w^2}{2^m}$. Thus, the LS estimator can provide a more precise estimate for the rank-p channel estimator than in the single transmit antenna case.

IV Simulation Results and Discussion

We use 4 transmit antennas and 1 receive antenna, with QPSK modulation where one frame is assumed to have 300 super blocks (each super block contains the 4 OFDM symbols to be transmitted from 4 transmit antennas at the same time). 1000 independent channel realizations are used to obtain the simulation results. This OFDM system operates with a bandwidth of 20MHz, so the sampling interval T_s=50ns. The bandwidth is divided into N= 64 tones and we assume that the root-mean square (rms) delay spread of the multipath channel is $\tau_{rms} = 1T_s$. In addition, cyclic prefix length L=16 is used.

First, the MSE of low rank channel estimation by SVD is investigated.

The actual MSE is calculated using $MSE = \frac{1}{4N}\sum_{i=1}^{4}\|H_i - H_{ip}\|^2$ and is compared with the theoretical MSE of rank-p channel estimator from [1] suitably modified for QPSK modulation and K ($2^{m-1} < K \le 2^m$) transmit antennas as described.

$$MSE(p) = \frac{1}{N}\sum_{i=1}^{p}(\lambda_i(1-\delta_i)^2 + \frac{K\sigma_w^2}{2^{m+1}}) + \frac{1}{N}\sum_{i=p+1}^{N}\lambda_i \quad (25)$$

Fig.2 shows that the simulated and theoretical MSEs are very close to each other when the value p is chosen a little larger than the length of the channel.

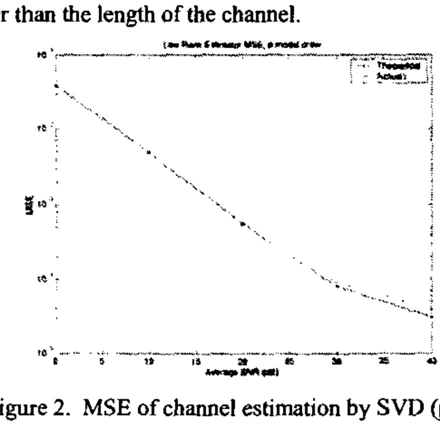

Figure 2. MSE of channel estimation by SVD (p=7)

Second, we compare the BER performance when the channel is estimated by SVD with the BER performance when the channel is known at the receiver (Figure.3). The two curves are very close to each other for p=7, but diverge for p=5 high SNR. The reason for this performance degradation for p=5 (rank equal to the minimal value) is attributable to loss of signal information when the SNR is high. Moreover, comparison between the performances of Alamouti's STBC and the rate-1 Non-orthogonal STBC shows that the latter can provide better BER. The Non-orthogonal STBC with p=7 even outperforms Alamouti's STBC with full channel knowledge at the receiver. Since the channel estimation problem can be solved at low complexity, it is concluded that the rate-1 Non-orthogonal STBC is promising for high-rate transmission over MIMO frequency selective channels.

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Appendix

Definition of p-canonical set [2]

A set of p real square matrices, each of size p, is a p-canonical set if the nonzero matrix entries equal one, I_p belongs to the set, and the matrices verify the follow properties.

1) $C_k^H C_l = C_l^H C_k = (C_k^H C_l)^H, \forall k, l = 1, 2, \cdots, p$

2) C_k has q nonzero entries, located in the first q rows, and D_k has (p-q) nonzero entries, located in rows q+1 through p; the ranks of C_k and D_k are q and (p-q), respectively.

3) Each matrix has at most one nonzero entry in each row and column.

4) No two matrices have nonzero entries in the same position (disjoint nonzero entries).

5) For a given k, C_k and D_k cannot have a nonzero entry on the same column,

6) A nonzero entry has modulus one (conservation of signal power)

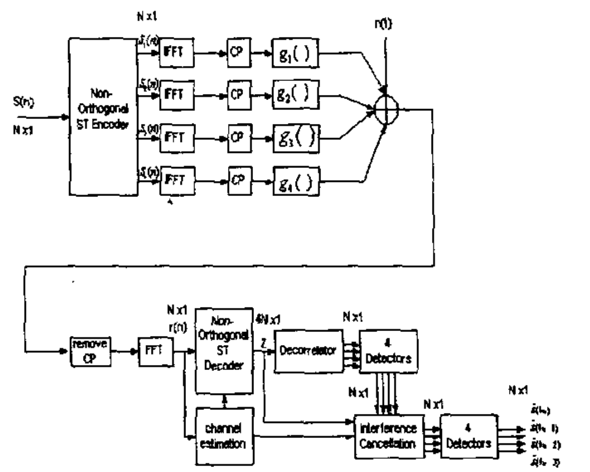

Figure 1 Baseband model of a Non-orthogonal ST coded ODFM system

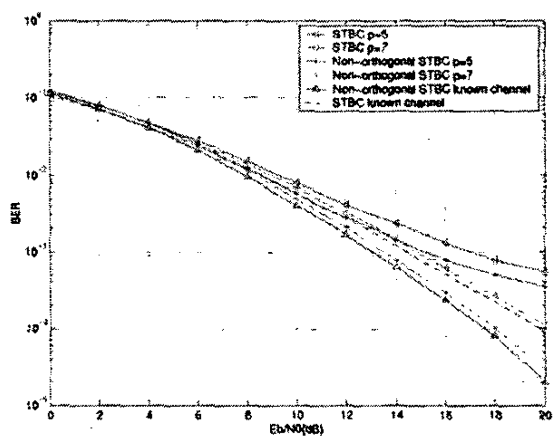

Figure 3. *BER performance comparison between* Non-orthogonal STBC and STBC