Low Complexity Blind Frequency-Offset Estimator for OFDM systems over ISI channels

Chengyang Li, Man-On Pun and Sumit Roy
Department of Electrical Engineering, BOX 352500
University of Washington, Seattle, WA 98195-2500
Email: {*cyli,mopun,roy*}*@ee.washington.edu*

Abstract—**Most previously reported carrier frequency offset (CFO) estimation algorithms for OFDM systems rely on the assumption of sufficient cyclic prefix (CP) - i.e. the channel length is less than the length of CP. In practice, this can be violated leading to significant performance degradation characterized by an irreducible error floor due to model mismatch. This paper focusses on CFO estimation for uncompensated ISI channels - it introduces a modified signal model and by exploiting the special structure of the filtering matrix due to CP and virtual carriers, a novel subspace CFO estimator is proposed. The method is attractive for its low complexity by avoiding SVD computation and potential to achieve high channel utilization by decreasing the length of CP in ISI channels. Preliminary computer simulations illustrate the effectiveness of the proposed algorithm.**

I. Introduction

Recently, Orthogonal Frequency-Division Multiplexing [1], [2] (OFDM) has attracted considerable research attention as a promising candidate for high data rate communications. It has already been adopted in various applications, for wired or wireless communications, e.g., ADSL, DAB/DVB-T, as well as IEEE 802.11a/HiperLAN.

Despite its many advantages, OFDM design faces several challenges. One critical issue is carrier frequency offset (CFO) due to frequency mismatch of the local oscillators between the transmitter and the receiver. This results in inter carrier interference (ICI) due to loss of orthogonality among the sub carriers and leads to significant error rate degradation [3]. The literature contains a large number of ideas devoted to CFO estimation based on data-aided or non-data-aided approaches. Non data-aided or *blind* estimators such as [4], [5], [6], are of special interest because of their potential to achieve higher channel utilization by omitting training data overhead. These methods exploit the redundant information in the channel output introduced at the transmitter by cyclic prefix (CP) [4] or virtual carriers (VC)[1] [5], [6].

Most reported CFO estimators make the assumption that CP is larger than the delay spread of the channel. However any chosen CP length may be rendered insufficient if used over larger delay spread wireless channels - this results in a model mismatch *vis-a-vis* the sufficient length CP assumption and consequent increases in the error rate, if left uncorrected. This motivates the work presented here - we focus on CFO estimation issue for OFDM systems over *ISI* channels (i.e., channels with length longer than CP) under the presumption that increasing CP is undesirable due to the loss in throughput. Specifically, we modify the signal model to take ISI channels into account. Modifying the method of [5] results in a new subspace based CFO estimator by exploiting the presence of both CP and VC. The proposed estimator is attractive for its higher channel utilization[2] and low complexity: it is capable of dealing with ISI channels without increasing CP where previous methods will fail. The algorithm exploits the special structure of the OFDM modulation and does not involve expensive (and sometimes prohibitive) SVD computation, despite being a subspace based approach.

Rest of the paper is organized as follows: A modified baseband signal model for the OFDM system over ISI channels is introduced in Section II. Section III reveals the key structure of the filtering matrix, which leads to the proposed subspace channel estimator. Computer simulations are presented in Section IV to demonstrate the effectiveness of the algorithm. Finally, paper concludes in Section V with some remarks.

The notation used in this paper follows usual convention - vectors are denoted by symbols in boldface; $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ are complex conjugate, transpose and conjugate transpose of $(\cdot)$, respectively. $\|\cdot\|$ gives the Frobenius norm of matrix argument. $A(i_1 : i_2, j_1 : j_2)$ denotes a sub matrix obtained by extracting rows i_1 through i_2 and columns j_1 through j_2 from matrix A. If no specific range appears at the row or column position in notation

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[1]In practical FFT-based OFDM systems, a shaping filter is required to limit the transmitted signal spectrum to the desired band. In order to ease filter implementation, some sub carriers in the roll-off region, i.e. the band edge, of the filter are left unmodulated; these are referred to as virtual carriers. In IEEE 802.11a, 12 out of the total of 64 sub carriers are specified as virtual carriers

[2]Blind channel estimation methods that can be used for ISI channels have been proposed in recent papers (such as [7]). After CFO compensation and channel estimation, channel shortening filters [8] can be applied to keep the composite channel length within the given CP, thus assuring the validity of the conventional per-tone equalization schemes.

$A(i_1 : i_2, j_1 : j_2)$ (e.g., $A(:, j_1 : j_2)$ or $A(i_1 : i_2, :)$), then all rows or columns are accounted for constituting the sub matrix.

II. Signal Formulation

In this section, we develop a modified received signal model for OFDM systems over *ISI channels*. The signal model *without* CFO is described first, followed by one where CFO is incorporated.

A. In absence of CFO

Let us consider an OFDM system (see Fig. 1) with Q sub carriers, of which only P are modulated by user's data symbols; the remaining $Q-P$ unmodulated carriers constitute *virtual carriers*. Assume the sub carriers numbered p_0 to p_0+P-1 are used for data (p_0 is the index of the first data carrier) and that the length of CP is D. Let the kth block of the 'frequency domain' information symbols be

$$\mathbf{s}(k) = [s_0(k),\, s_1(k),\, \ldots,\, s_{P-1}(k)]^T. \tag{1}$$

Define a $Q \times Q$ IDFT matrix

$$\mathbf{W}_Q = \frac{1}{\sqrt{Q}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w_Q^{-1} & \cdots & w_Q^{-(Q-1)} \\ 1 & w_Q^{-2} & \cdots & w_Q^{-2(Q-1)} \\ \vdots & & & \vdots \\ 1 & w_Q^{-(Q-1)} & \cdots & w_Q^{-(Q-1)(Q-1)} \end{bmatrix} \tag{2}$$

with $w_Q = e^{-j2\pi/Q}$. Multi-carrier modulation, which is implemented by IFFT, yields the 'time domain' signal vector $[x_0(k), x_1(k), \ldots, x_{Q-1}(k)]^T = \mathbf{W}\,\mathbf{s}(k)$, where $\mathbf{W}$ is the $Q \times P$ partial IDFT matrix

$$\mathbf{W} = \begin{bmatrix} 1 & & & \\ & w_Q^{-p_0} & & \\ & & \ddots & \\ & & & w_Q^{-p_0(Q-1)} \end{bmatrix} \cdot \mathbf{W}_Q(:, 1:P). \tag{3}$$

CP insertion replicates the last D elements of the IFFT output vector in the front and results a $J \times 1$ ($J = Q + D$) OFDM symbol vector

$$\begin{aligned} \mathbf{x}(k) &= [x_{Q-D}(k), \ldots, x_{Q-1}(k), x_0(k), x_1(k), \ldots, x_{Q-1}(k)]^T \\ &= \underbrace{\begin{bmatrix} \mathbf{W}(Q-D+1:Q,:) \\ \mathbf{W} \end{bmatrix}}_{\bar{\mathbf{W}}} \mathbf{s}(k) \\ &= \bar{\mathbf{W}}\,\mathbf{s}(k). \end{aligned}$$

Each element or 'chip' in the vector $\mathbf{x}(k)$ is then pulse shaped by $g_{tr}(t)$ to generate the continuous time signal sent on the channel

$$x(t) = \sum_{k=-\infty}^{+\infty} \sum_{p=0}^{J-1} x_p(k) g_{tr}(t - (p + kJ)T), \tag{4}$$

where T is the chip period. Thus denoting $q = p + kJ$, we identify $k = \lfloor \frac{q}{J} \rfloor$ ($\lfloor x \rfloor$ is the largest integer contained in x) and $p = q$ *modulo* J. Then the transmitted signal $x(t)$ can be rewritten as

$$x(t) = \sum_{q=-\infty}^{+\infty} x_q g_{tr}(t - qT) \tag{5}$$

where x_q is an equivalent transmitted 'chip' sequence corresponding to the $x_p(k)$'s.

During the transmission, the signal $x(t)$ passes through a dispersive channel with impulse response $c(t)$, is contaminated by AWGN noise $n(t)$, and finally is input into a front-end receive filter $g_{rx}(t)$.

Defining the composite channel filter $h(t) = g_{tr}(t) * c(t) * g_{rx}(t)$ and the filtered noise $v(t) = n(t) * g_{rx}(t)$ where $*$ denotes linear convolution, the received signal $r(t)$ is therefore

$$r(t) = \sum_{q=-\infty}^{+\infty} x_q h(t - qT) + v(t) \tag{6}$$

Assume the composite channel $h(t)$ to have finite support $[0, LT]$ where $D < L < J$, corresponding to the scenario that the channel delay spread is longer than the CP but does not exceed the OFDM symbol duration[3]. A synchronized rate $1/T$ sampler after $r(t)$ yields (t_0 denotes the sampling phase)

$$r(i) = r(t_0 + iT) = \sum_{l=0}^{L} x_{i-l} h(t_0 + lT) + v(i) \tag{7}$$

where $v(i) = v(t_0 + iT)$.

Let $h(l) = h(t_0 + lT)$ and the channel vector

$$\mathbf{h} = [h(L),\, h(L-1),\, \ldots,\, h(0)]^T. \tag{8}$$

Define an $J - L \times J$ *Toeplitz* matrix $\mathcal{H}$ constructed from $\mathbf{h}$

$$\begin{aligned} \mathcal{H} &= Toeplitz([\mathbf{h}^T \; \underbrace{0 \ldots 0}_{(J-L-1)\ 0's}\;]) \\ &= \begin{bmatrix} h(L) & \ldots & h(0) & & & & \\ & h(L) & \ldots & h(0) & & & \\ & & \ddots & & \ddots & & \\ & & & h(L) & \ldots & h(0) \end{bmatrix} \end{aligned} \tag{9}$$

[3]ISI restricted to the past neighboring symbol is generally true for typical OFDMs

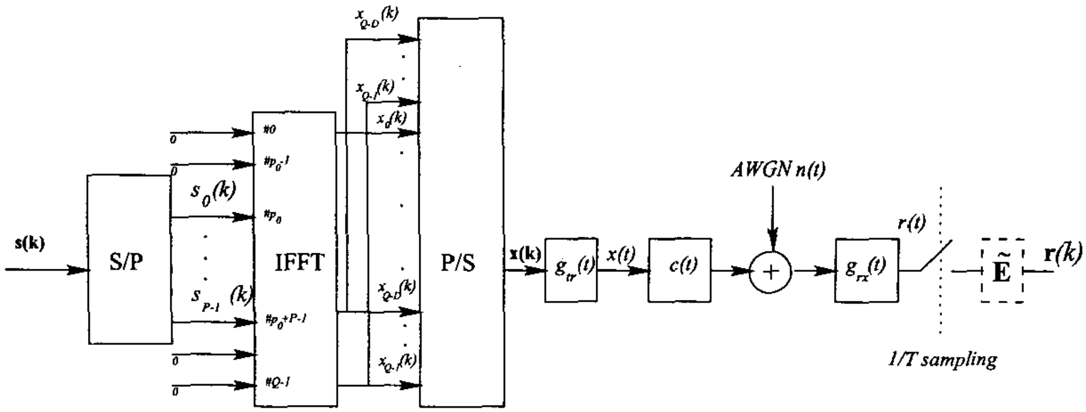

Fig. 1. Baseband model for OFDM system with both CP and VC (dashed box $\tilde{\mathbf{E}}$ illustrates the impact of CFO when presents)

Collecting *only* the ISI-free samples corresponding to the K^{th} OFDM symbol results in a $J-L\times 1$ received signal vector

$$\begin{aligned}\mathbf{r}(k) &= [r(kJ+L),\ldots,r((k+1)J-1)]^T \\ &= \mathcal{H}\bar{\mathbf{W}}\mathbf{s}(k)+\mathbf{n}(k). \end{aligned} \quad (10)$$

B. In presence of CFO

Now assume there is carrier frequency mismatch, ϕ, between the transmitter and the receiver, where $\phi = 2\pi\Delta fT$ is the normalized CFO with Δf being the frequency offset in Hz. Due to the presence of CFO, the signal part of $\mathbf{r}(k)$ in (10) is modulated by

$$\tilde{\mathbf{E}} = \underbrace{diag([e^{j\phi 0}, e^{j\phi},\ldots,e^{j\phi(J-L-1)}])}_{\mathbf{E}} \cdot e^{j\phi[(k-1)J+L]}$$

and yields

$$\mathbf{r}(k) = \mathbf{E}\underbrace{\mathcal{H}\bar{\mathbf{W}}}_{\mathcal{A}}\mathbf{s}(k)e^{j\phi((k-1)J+L)}+\mathbf{n}(k). \quad (11)$$

With the *modified* signal model (11), our goal is to estimate the CFO from the observation $\mathbf{r}(k)$ without any training data.

III. Blind CFO Estimation

We remark that the filtering matrix $\mathcal{A}$ is highly structured as revealed next; we then proceed to a new subspace based CFO estimator based on this structure.

A. Special structure of the filtering matrix $\mathcal{A}$

Let the frequency response of the channel coefficients [4] $[h(0),\ldots,h(L)]^T$ on the active data carriers be the $P\times 1$ vector

$$\begin{aligned}\mathbf{H} &= [H(p_0),\ldots,H(p_0+P-1)]^T \\ &= (\mathbf{W}(1:L+1,:))^H \begin{bmatrix} & & & 1 \\ & & \cdot & \\ & \cdot & & \\ 1 & & & \end{bmatrix} \mathbf{h} \end{aligned} \quad (12)$$

A 'rotated' version of the frequency response denoted by $\mathbf{g}$ is given by

$$\begin{aligned}\mathbf{g} &= [g(0),\ldots,g(P-1)]^T \\ &= \begin{bmatrix} w_Q^{-p_0L} & & \\ & \ddots & \\ & & w_Q^{-(p_0+P-1)L} \end{bmatrix}\mathbf{H} \\ &= \begin{bmatrix} h(L)+\ldots+h(0)w_Q^{-p_0L} \\ \vdots \\ h(L)+\ldots+h(0)w_Q^{-(p_0+P-1)L} \end{bmatrix}\end{aligned}$$

The structure of the matrices $\mathcal{H}$ (Toeplitz) and $\bar{\mathbf{W}}$ allows the first $J-L$ elements in the ith ($i=1,\ldots,P$) column of $\mathcal{A}$ to be

$$\begin{aligned}\mathcal{A}(:,i) &= \mathcal{H}\bar{\mathbf{W}}(:,i) \\ &= \begin{bmatrix} w_Q^{-(p_0+i-1)(Q-D)}(h(L)+\ldots+h(0)w_Q^{-(p_0+i-1)L}) \\ \vdots \\ w_Q^{-(p_0+i-1)(Q-L-1)}(h(L)+\ldots+h(0)w_Q^{-(p_0+i-1)L}) \end{bmatrix} \\ &= \bar{\mathbf{W}}(:,i)\cdot g(0)\end{aligned}$$

Hence, it is easy to see the following relation holds, which connects the channel 'rotated' frequency response vector $\mathbf{g}$ to the filtering matrix $\mathcal{A}$:

$$\mathcal{A} = \mathcal{H}\bar{\mathbf{W}}$$

[4]Note that the channel vector $\mathbf{h}$ is defined throughout as $[h(L),\ldots,h(0)]^T$

$$= \sqrt{Q}\bar{\mathbf{W}}(1:J-L,:)\underbrace{\begin{bmatrix} g(0) & & \\ & \ddots & \\ & & g(P-1) \end{bmatrix}}_{\mathbf{G}}$$

$$= \sqrt{Q}\bar{\mathbf{W}}(1:J-L,:)\mathbf{G} \quad (13)$$

Substituting (13) into the modified signal model (11) yields

$$\mathbf{r}(k) = \sqrt{Q}\mathbf{E}\bar{\mathbf{W}}(1:J-L,:)\mathbf{G}\mathbf{s}(k)e^{j\phi((k-1)J+L)} + \mathbf{n}(k). \quad (14)$$

B. Subspace based CFO Estimator

The structure revealed in (14) is the key leading to the proposed CFO estimator. Note that the $J-L \times P$ matrix $\bar{\mathbf{W}}(1:J-L,:)$ is constructed by keeping the first $J-L$ rows of $\bar{\mathbf{W}}$. $\bar{\mathbf{W}}(1:J-L)$ is a *tall* matrix when

$$J - L > P. \quad (15)$$

and can be shown to be full column rank[5]. Its SVD asserts

$$\bar{\mathbf{W}}(1:J-L,:) = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_s \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \quad (16)$$

where $[\mathbf{U}_s, \mathbf{U}_n]$ is an $J-L \times J-L$ unitary matrix. The P columns of $\mathbf{U}_s$ reside in the same column space of $\bar{\mathbf{W}}(1:J-L,:)$, while $J-P-L$ column vectors of $\mathbf{U}_n$ span the subspace orthogonal to $\mathbf{U}_s$. $\mathbf{\Sigma}_s = diag(\lambda_1, \lambda_2, \ldots, \lambda_P)$ is a diagonal matrix consisting of P non-zero singular values. If there's no CFO, by the property of orthogonality,

$$\mathbf{U}_n^H \bar{\mathbf{W}}(1:J-L,:)\mathbf{G}\mathbf{s}(k)e^{j\phi((k-1)J+L)} = \mathbf{0}. \quad (17)$$

Therefore, in the spirit of [5], by defining $\mathbf{Z} = diag(e^{j\theta 0}, \ldots, e^{j\theta(J-L-1)})$, for noise free situation, CFO *can be determined by solving for θ from the equation*

$$\mathbf{U}_n^H \mathbf{Z}^H \mathbf{r}(k) = 0. \quad (18)$$

In presence of noise, an appropriate cost function after receiving K OFDM symbols is given by

$$C(\theta) = \sum_{k=1}^{K} \|\mathbf{U}_n^H \mathbf{Z}^H \mathbf{r}(k)\|^2, \quad (19)$$

and the proposed CFO estimator is then

$$\hat{\phi} = \arg\min_{\theta} C(\theta). \quad (20)$$

Remarks:

1. The observation of the structure revealed in (13) is crucial to the consequent estimator (20).

[5]Due to space limitations, the proof is not included and deferred to a fuller version.

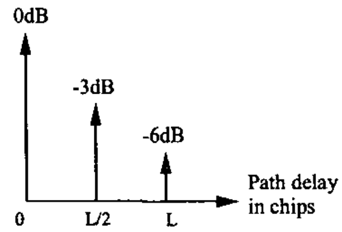

Fig. 2. Delay profile of the channel

2. The modified signal model (14) holds for channels with length $L < J$. For the proposed method to work, it is necessary that the sum of VC and CP is larger than the channel order L (15). The use of VC enables the CFO estimator to perform adequately over ISI channels unlike the estimators such as [5] (unless CP is increased). From another perspective, the proposed estimator illustrates the possibility of decreasing the length of CP in OFDM systems that precipitates insufficient CP condition. We note that blind channel estimators (e.g. [7]) have been proposed that are able to estimate ISI channels for insufficient CP scenarios. Furthermore, channel shortening schemes (e.g. [8]) can be used to constrain the composite channel within the shortened CP length, thus preserving the conventional simple per-tone equalization schemes.

3. *Another merit of the estimator is its low computation* complexity. Though a subspace based approach, it avoids the expensive SVD computation of the observed data vectors. Instead, the subspace $\mathbf{U}_n$ orthogonal to the signal subspace is determined directly by $\bar{\mathbf{W}}(1:J-L,:)$. Therefore, once L (that need not be the accurate channel order, since any upper bound suffices) is specified, $\mathbf{U}_n$can be calculated off-line.

IV. SIMULATION RESULTS

Preliminary computer simulations were conducted to assess the effectiveness of the proposed estimator with comparison to [5], when ISI channel presents. For comparison reasons, we consider the *same* OFDM system used in [5] with the following system parameters: $Q = 32$, $P = 20$, $p_0 = 0$, $K = 4$ and $\phi = 3.67\Delta\omega$, where $\Delta\omega = 2\pi/N$ is the channel spacing. In our simulations, we use $D = 8$ and a deterministic 3-path channel as shown in Fig.2. To measure the estimators' performance, normalized mean square error (NMSE) defined as

$$\text{NMSE} = E\left[\left(\frac{\hat{\phi}-\phi}{\Delta\omega}\right)^2\right] \quad (21)$$

is used.

In order to highlight the proposed estimator's capability of handling ISI channels, we set the channel order to be

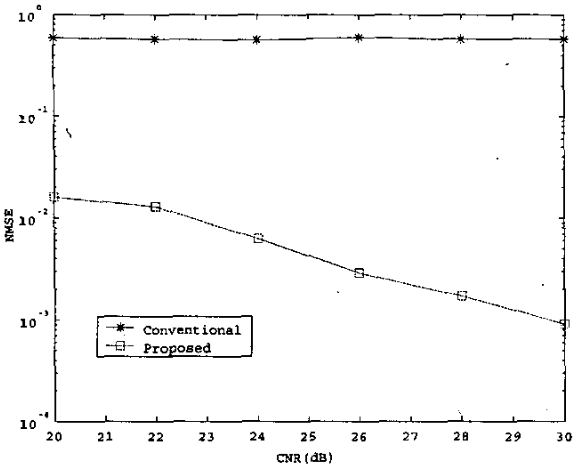

Fig. 3. Estimation performance $L = 12, D = 8$

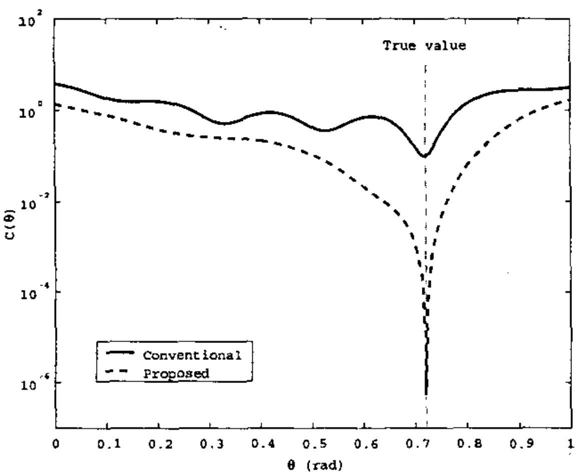

Fig. 4. Cost function comparison

$L = 12$. Fig.3 shows the simulation results over 200 realizations. By taking ISI into account in the modified signal model (as in Eq.(14)), the proposed estimator achieved better estimation performance than the conventional estimator in [5].

More insight can be obtained by comparing the two cost functions (Eq.(19) for the proposed and Eq.(4) in [5]) in noise-free environment, as shown in Fig.4. The proposed cost function has a single, deep minimum at the true CFO unlike the conventional in [5], and thus results in its better performance.

V. Conclusion

Most previously reported CFO estimation algorithms rely on the assumption of sufficient CP, i.e., the channel length is less than the length of CP. However, in some scenarios, the above assumption can be violated. In presence of ISI channels, those estimators will suffer an irreducible error floor because of model mismatch and consequently cause OFDM systems significant degradation of bit error rate performance. In this paper, we introduced a modified signal model which considers ISI channels. By exploiting the special structure of the filtering matrix due to CP and VC, a novel subspace CFO estimator is proposed. The method is attractive for its low complexity and potential for OFDM system to achieve high channel utilization by decreasing the length of CP. Preliminary computer simulations have illustrated the effectiveness of the proposed algorithm.

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