Detection and Imaging of Objects Behind Multiple Scattering Random Obscuring Layers

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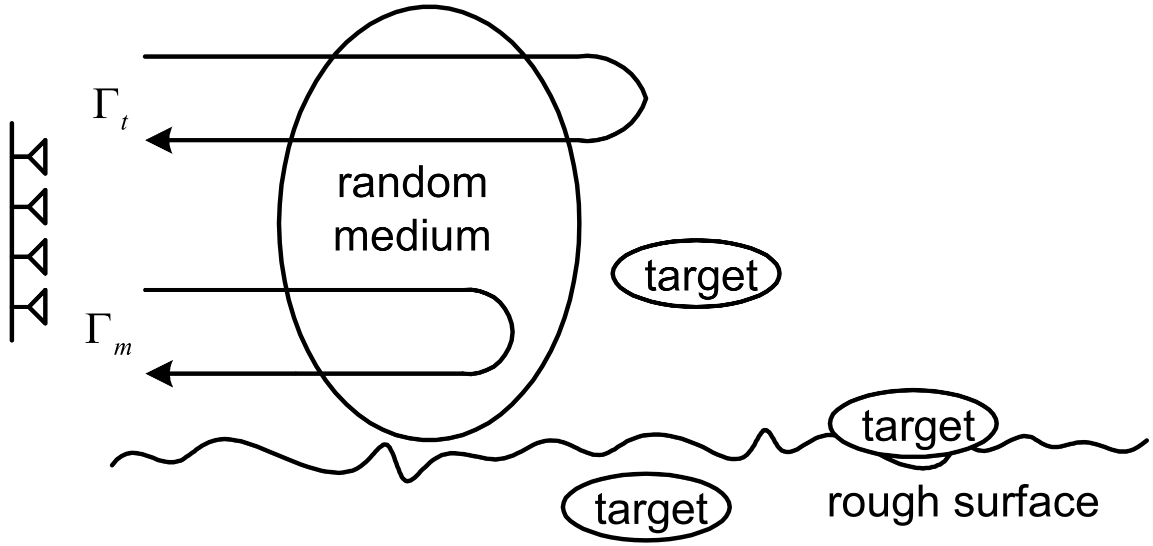
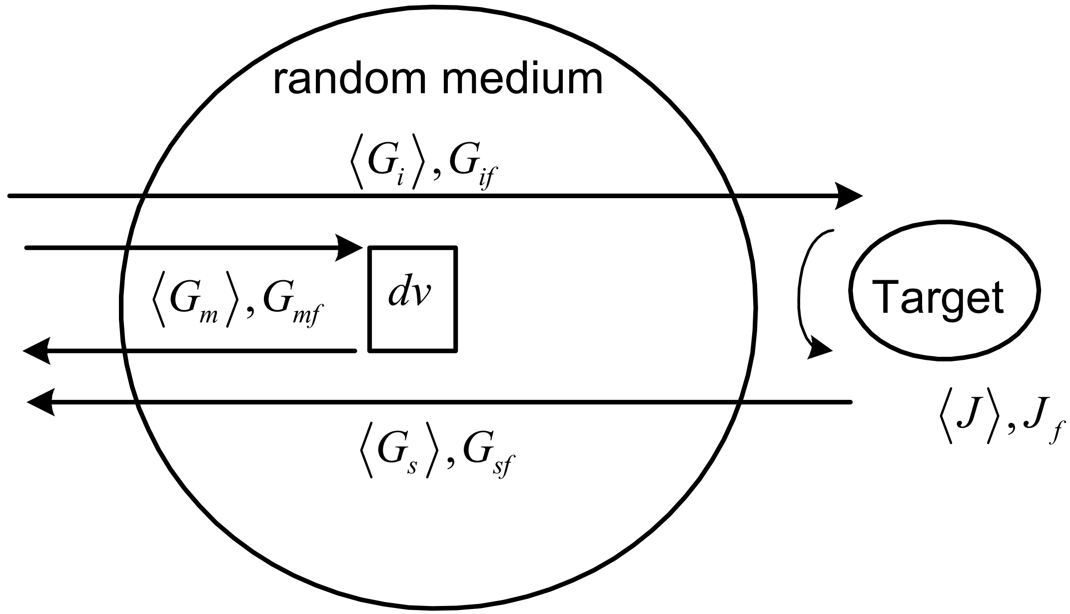
Object detection and imaging in random medium – general problems



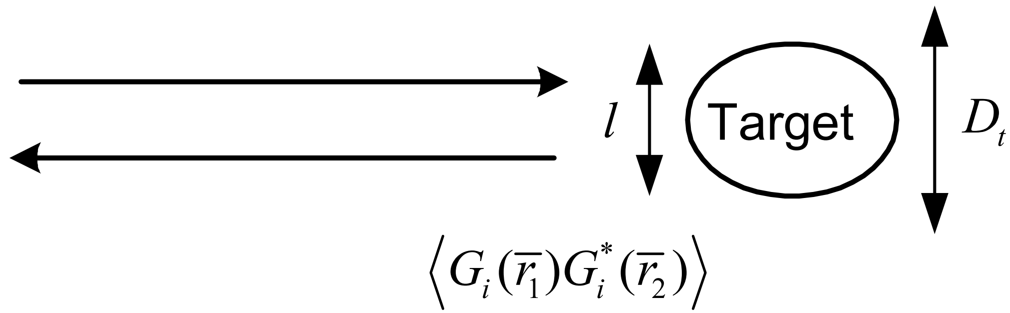
- Array or aperture
- UWB: Two-frequency MCF (Mutual coherence function)
- Focusing: Resolution, spatial and temporal
- $\Gamma_m(t) = \text{medium and rough surface} = \langle \psi_m \psi_m^* \rangle$
  $\Gamma_t(t) = \text{target scattering} = \langle \psi_t \psi_t^* \rangle$

- Stochastic Green's function $\langle G \rangle + G_f$
- Stochastic surface currents $\langle J \rangle + J_f$



$\langle G_i \rangle + G_{if} =$ incident Green's function
$\langle J \rangle + J_f =$ surface current on object (stochastic integral equation)
$\langle G_s \rangle + G_{sf} =$ scattered wave
$\langle G_m \rangle + G_{mf} =$ medium scattering

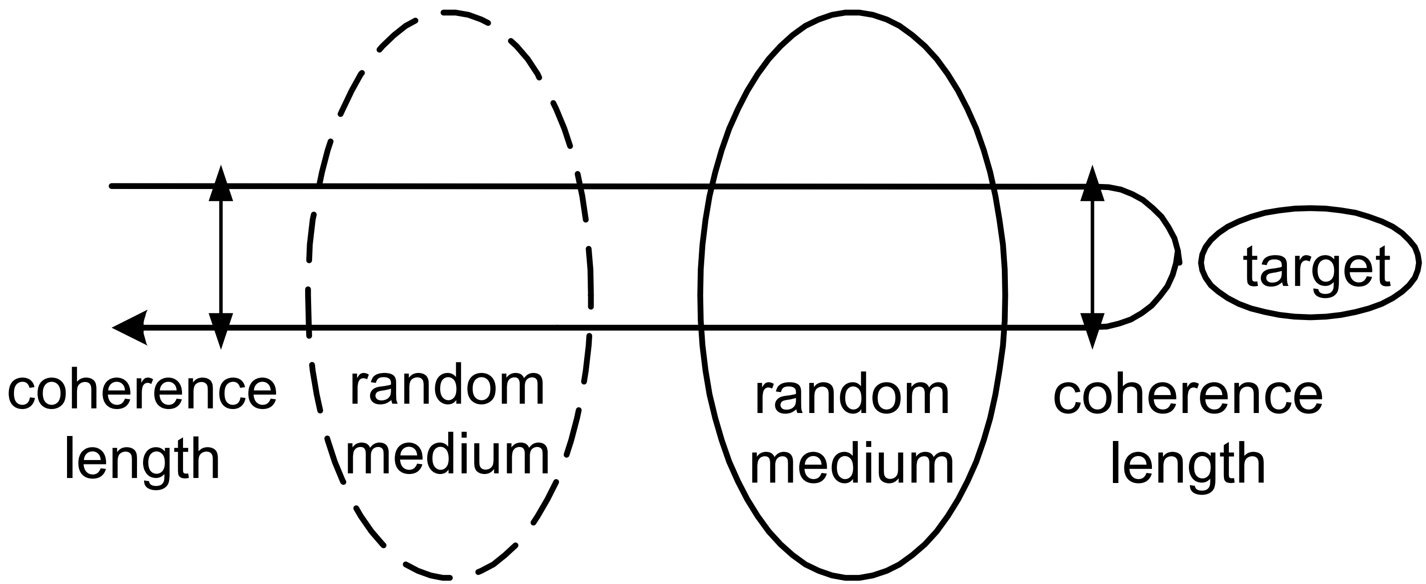
- Coherence length and target size



$$l = \text{ coherence length}$$
$$D_t = \text{ target size}$$

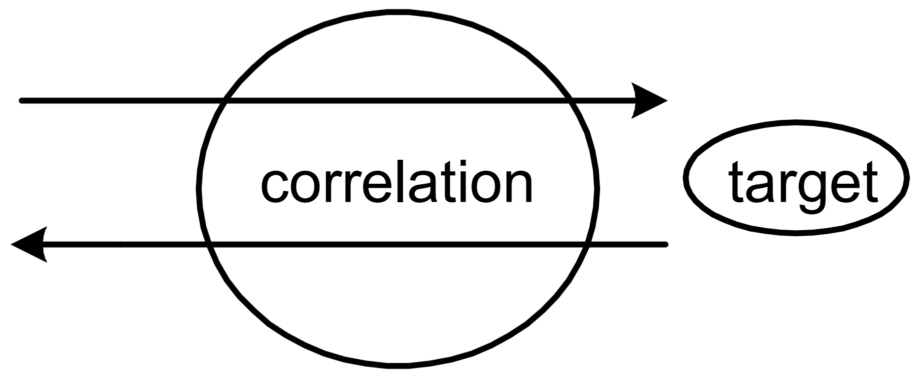
- $l \gg D_t \rightarrow$ plane wave, scattering amplitude (point scatterer)
  $l \ll D_t \rightarrow$ coherence length is important (Kirchhoff approximation)

- Shower curtain effect (location of random medium)



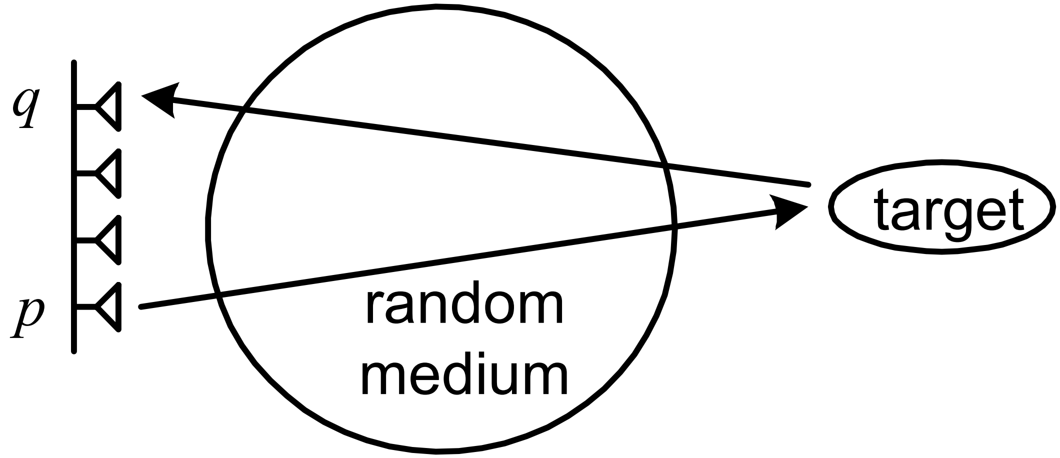
- Higher resolution if random medium is closer to target

- Backscattering enhancement



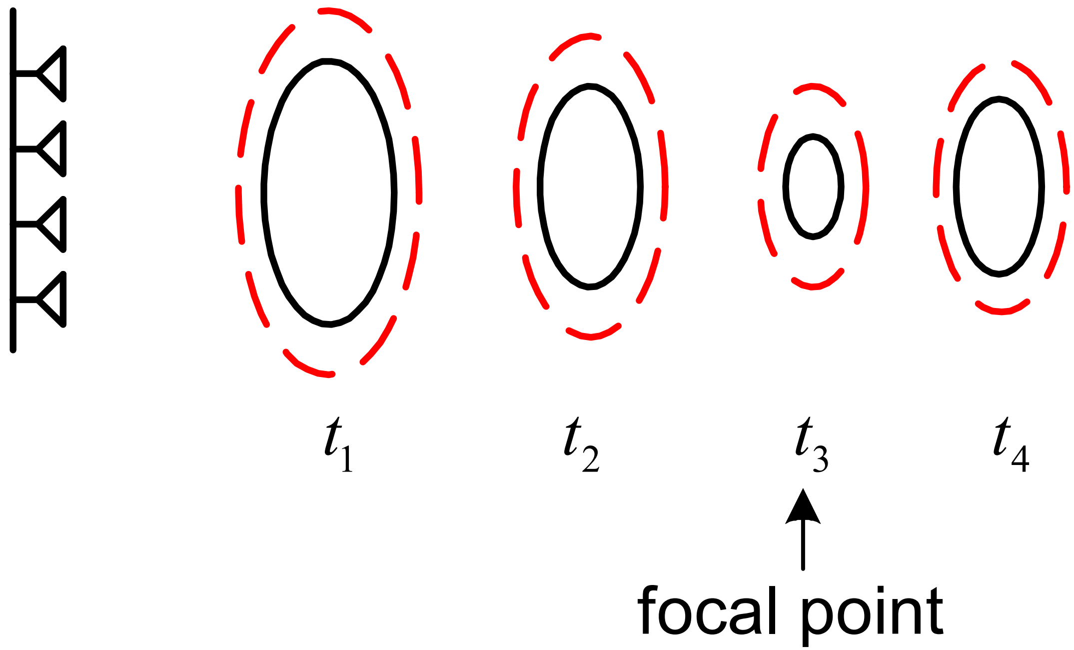
- Increase of backscattered power due to correlation between forward and backward waves

- Focusing and tomographic imaging multiple transmitters and receivers



$$p = N \times N \ \ \text{transmitters}$$
$$q = N \times N \ \ \text{receivers}$$

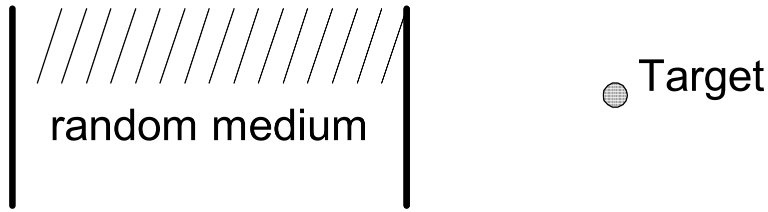
- UWB in random medium

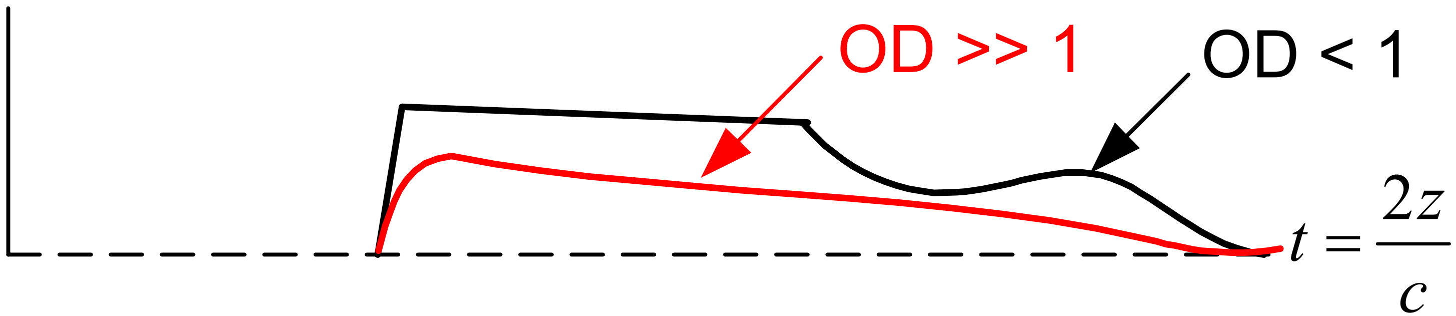


Black in free space

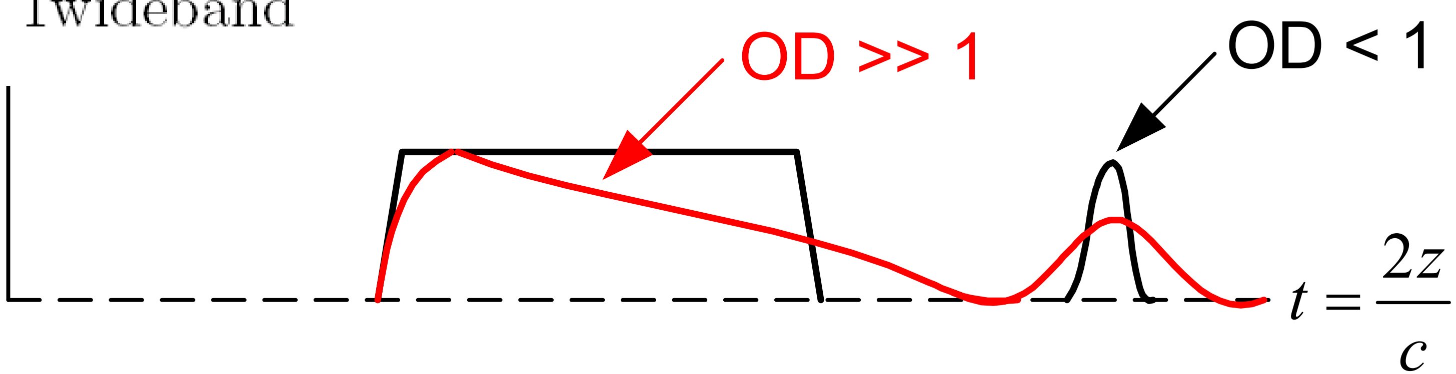
Red in random medium

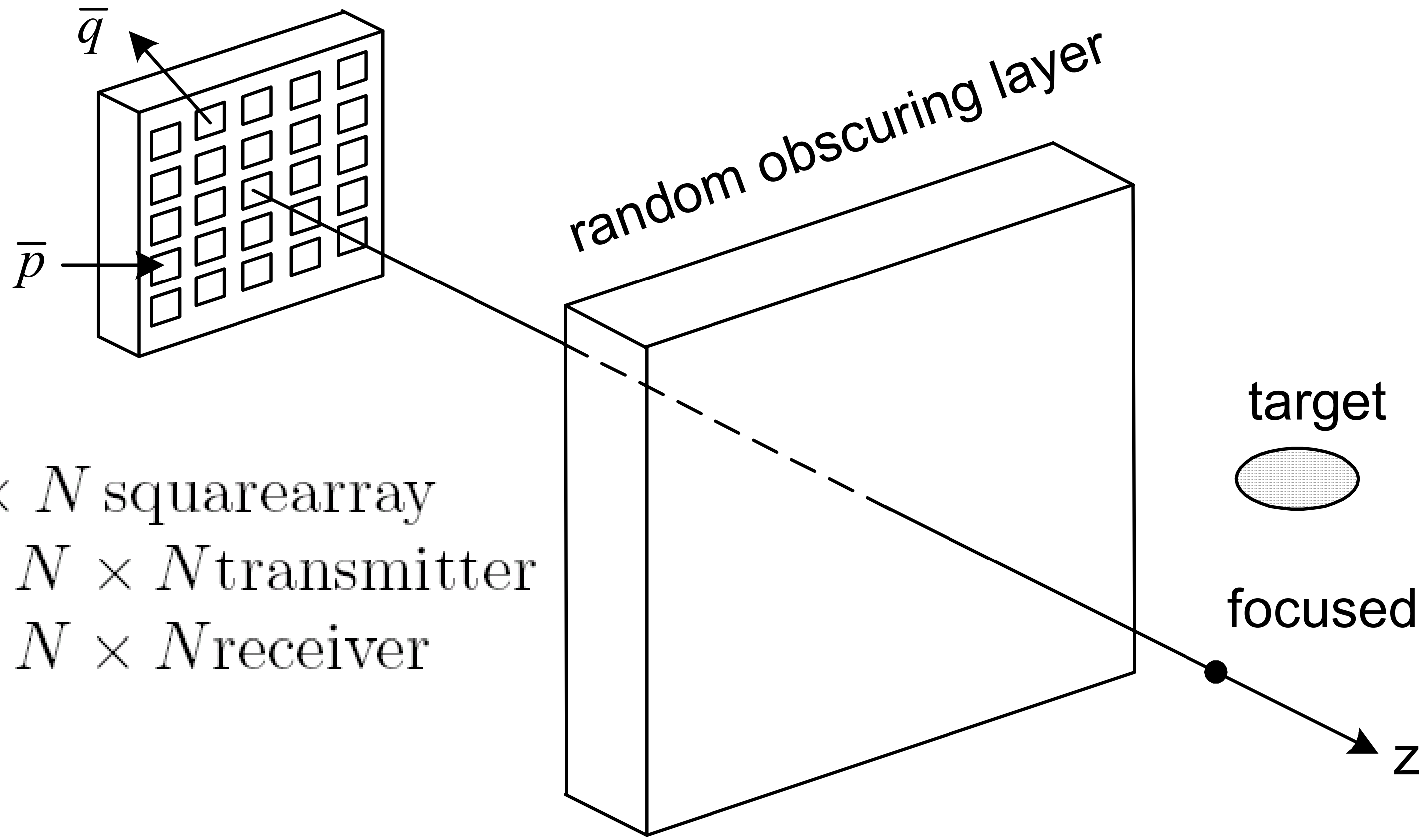
- Time response
- $\frac{\Delta\omega}{\omega_o} \ll 1$ narrow band





- $\frac{\Delta\omega}{\omega_o} < 1$wideband





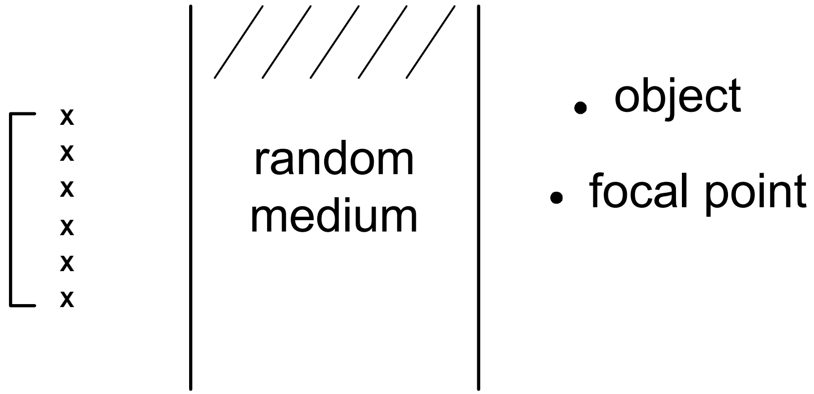
- $N \times N$ square array
  $\bar{p} = N \times N$ transmitter
  $\bar{q} = N \times N$ receiver
- UWB pulse
- Focused – transmitter and receiver

Outline

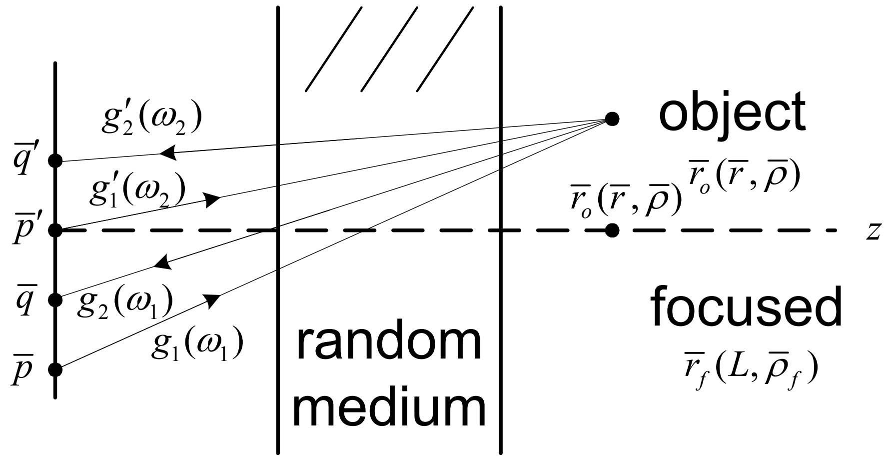
1. Problem statement
2. General formulations
3. Generalized two-frequency mutual coherence function (MCF)
4. General solution for imaging
5. Approximate analytical solutions
6. Final expression for $\Gamma(t, z, \rho)$
7. Resolution, coherence length, shower curtain effects, backscattering enhancement
8. Other related problems:
   - Objects in random media
   - Objects near rough surfaces

1. Problem statement

- Transmitter – Receiver
  - Array or aperture – focused
  - UWB pulse
- General Formulations and solutions two-frequency MCF
- Analytical solution
- Effects
  - Resolution
  - Coherence length, coherence time
  - Shower curtain effects (location of random medium)
  - Backscattering enhancement
- Relations
  - SAR, Confocal imaging
  - OCT (Optical coherence tomography)
  - MIMO (Multiple-input multiple-output)
- Other related problems



2. General formulations



$$\Gamma(t_1,t_2) = \text{Output}$$

$$= \frac{1}{2\pi}\int\int d\omega_1\, d\omega_2\, \exp\left(-i\omega_1 t + i\omega_2 t\right)\Gamma(\omega_1,\omega_2)F(\omega_1)F^*(\omega_2)$$

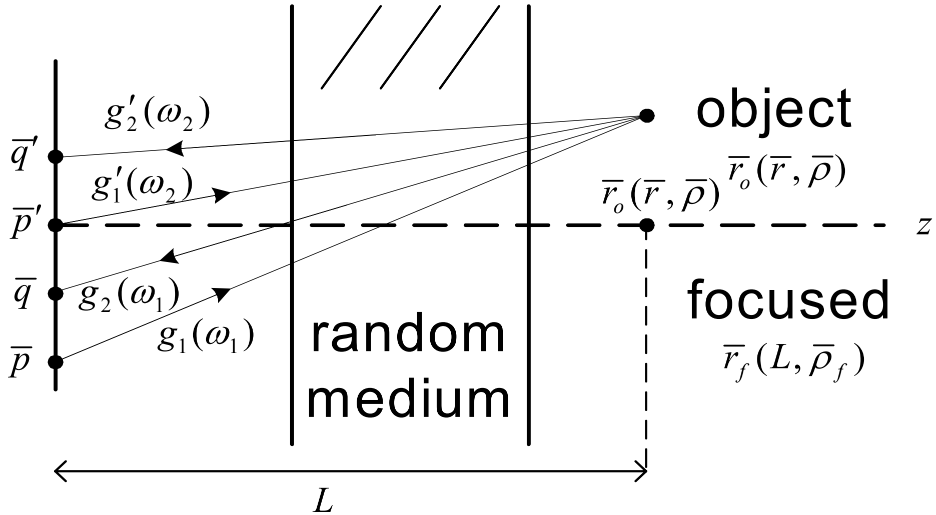
$$\Gamma(\omega_1,\omega_2) = \text{Two frequency mutual coherence function - }4^{\text{th}}\text{ order}$$

$$= \sum_p \sum_q \sum_{p'} \sum_{q'} \langle g_1(\omega_1) g_2(\omega_1) g_1'^*(\omega_2) g_2'^*(\omega_2)\rangle$$

$$F(\omega_1) = \text{Transmitter spectrum}$$

$$g_1, g_2, g_1', g_2' = \text{Stochastic Green's functions}$$

General formulations - stochastic Green's functions



$$g_1 = G_1(\bar{p},\bar{r}_o,\omega_1)\, u(\bar{p},\bar{r}_f,\omega_1)$$

focusing function

Stochastic Green's function (forward) at ω_1

$$g_2 = G_2(\bar{q},\bar{r}_o,\omega_1)\, u(\bar{q},\bar{r}_f,\omega_1) = \text{backward at } \omega_1$$

$$g_1' = G_1'(\bar{p}',\bar{r}_o,\omega_2)\, u(\bar{p}',\bar{r}_f,\omega_2) = \text{forward at } \omega_2$$

$$g_2' = G_2'(\bar{q}',\bar{r}_o,\omega_2)\, u(\bar{q}',\bar{r}_f,\omega_2) = \text{backward at } \omega_2$$

General formulations - Gaussian modulated pulse

- pulse

$$f(t) = \exp\left(-\frac{t^2}{T_o^2} - i\omega_o t\right)$$

$$F(\omega_1) = \int f(t)\exp(+i\omega_1 t)dt$$

$$= \sqrt{\pi}\,T_o \exp\left(-\frac{(\omega-\omega_o)^2}{\Delta\omega^2}\right), \quad \Delta\omega = \frac{2}{T_o}$$

- Coherence function $\Gamma(t_1 = t_2 = t)$

$$= \frac{1}{(2\pi)^2}\int\int d\omega_c\, d\omega_d \exp(-i\omega_d t)\Gamma(\omega_1,\omega_2)F(\omega_1)F(\omega_2), \quad \omega_c = (\omega_1+\omega_2)/2,\, \omega_d = \omega_1 - \omega_2$$

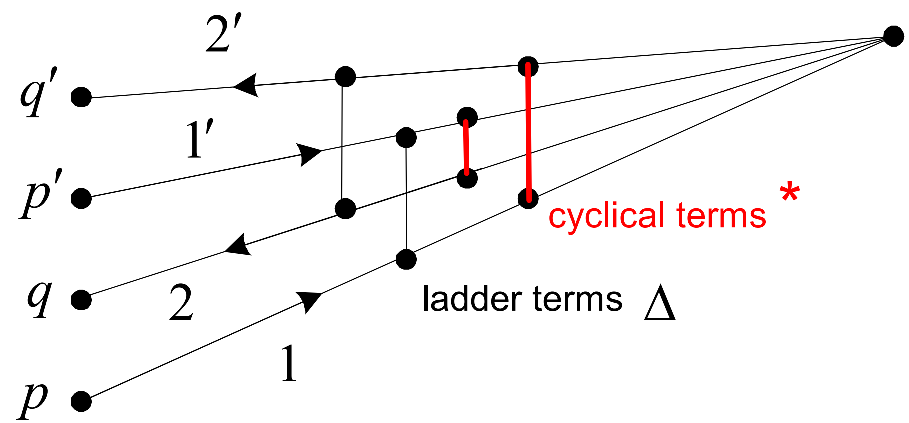
$$= \frac{1}{\pi}\frac{1}{\Delta\omega}\sqrt{\frac{\pi}{2}}\int d\omega_d\, \exp(-i\omega_d t)\Gamma(\omega_d)\exp\left(-\frac{\omega_d^2}{2\Delta\omega^2}\right), \quad \Gamma(\omega_1,\omega_2) = \text{slowly varying function of } \omega_c$$

$$\Gamma(\omega_d) = \sum_p\sum_q\sum_{p'}\sum_{q'}\langle g_1 g_2 g_1'^* g_2'^*\rangle = \text{fourth order moments}$$

General formulation – Calculation of fourth order moments

$$\Gamma(\omega_d) = \langle g_1 g_2 g_1'^* g_2'^* \rangle$$
$$= \langle g_1\rangle\langle g_2\rangle\langle g_1'^*\rangle\langle g_2'^*\rangle$$
$$+\langle g_1\rangle\langle g_1'^*\rangle\langle g_{2f} g_{2f}'^*\rangle \quad \Delta$$
$$+\langle g_2\rangle\langle g_2'^*\rangle\langle g_{1f} g_{1f}'^*\rangle \quad \Delta$$
$$+\langle g_{1f} g_{1f}'^*\rangle\langle g_{2f} g_{2f}'^*\rangle \quad \Delta$$
$$+\langle g_1\rangle\langle g_2'^*\rangle\langle g_{2f} g_{1f}'^*\rangle \quad *$$
$$+\langle g_2\rangle\langle g_1'^*\rangle\langle g_{1f} g_{2f}'^*\rangle \quad *$$
$$+\langle g_{1f} g_{2f}'^*\rangle\langle g_{2f} g_{1f}'^*\rangle \quad *$$

Use circular complex Gaussian assumption

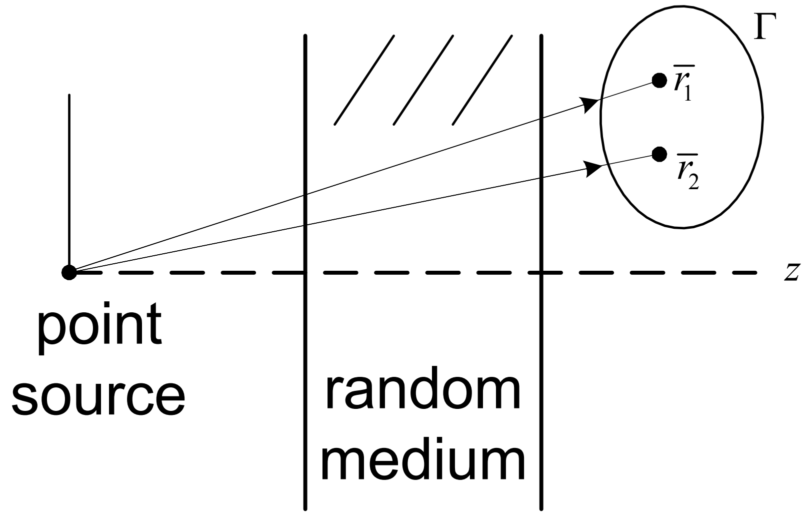


Two-frequency:

$$g_1(\omega_1), g_2(\omega_1), g_1'(\omega_2), g_2'(\omega_2)$$
$$g_1(\omega) = \langle g_1\rangle + g_{1f} \text{ (fluctuation)}$$

- Need to consider
  - coherent field $\langle g_1(\omega_1)\rangle$
  - incoherent two-frequency MCF $\langle g_{1f}(\omega_1) g_{1f}'^*(\omega_2)\rangle$

3. Two-frequency MCF for point source



- $\langle G \rangle = \text{coherent Green's function} = \langle G(\bar{r}, \omega) \rangle$
- $\Gamma = \text{Two-frequency MCF}$

$$= \Gamma(\bar{r}_1, \omega_1; \bar{r}_2, \omega_2)$$

$$= \langle \Gamma(\bar{r}_1, \omega_1) \Gamma^*(\bar{r}_2, \omega_2) \rangle$$

- $G = \text{Stochastic Green's function}$

- Two Frequency MCF
- Ishimaru, Sreenivasiah, Hong, Painter (1976, 1977, 1978, 1980)
  Oz, Heyman (1997)
  Bronshtein, Mazar et.al (2002)
  Young et.al (1996)
- Plane wave
- Modal theory
- Reference wave
- This paper was second order phase perturbation

Second order phase perturbation, Rytov, phase integral

$$\left[\bigtriangledown^2 + k^2\left(1+\epsilon_1\right)\right] G = -\delta(\bar{r})$$

- Born series

$$G = \sum_{n=0}^{\infty} G_n, \quad G_o(\bar{r}) = \text{unperturbed field in free space}$$

$$G_1(\bar{r}) = \int G(\bar{r},\bar{r}_1) k^2 \epsilon_1(\bar{r}_1) G_o(\bar{r}_1)\, dv_1,$$

$$G_2(\bar{r}) = \int\int G(\bar{r},\bar{r}_1) k^2 \epsilon_1(\bar{r}_1) G(\bar{r},\bar{r}_2) k^2 \epsilon_1(\bar{r}_2) G_o(\bar{r}_2)\, dv_1\, dv_2, \quad G_n = \mathcal{O}\left(\epsilon^n\right)$$

- Phase perturbation series

$$G(\bar{r}) = G_o(\bar{r}) \exp\left(\sum_{n=1}^{\infty} \psi_n\right)$$

- Equating the terms of the same powers of ϵ^n

$$\psi_1 = \frac{G_1}{G_o}, \quad \psi_2 = \frac{G_2}{G_o} - \frac{1}{2}\psi_1^2, \quad \psi_3 = \mathcal{O}\left(\epsilon^3\right)$$

Coherent field

$$\langle G\rangle = G_o\Big\langle \exp\left(\psi_1+\psi_2+\mathcal{O}(\epsilon^3)\right)\Big\rangle \qquad \Big\langle \exp(\psi)\Big\rangle = \exp\left[\langle\psi\rangle + \frac{1}{2}\langle(\psi-\langle\psi\rangle)^2\rangle\right]$$

$$\langle G\rangle = G_o \exp\left(\langle\psi_2\rangle + \frac{1}{2}\langle\psi_1^2\rangle\right)$$ but $$\psi_2 = \frac{G_2}{G_o} - \frac{1}{2}\psi_1^2 \quad .$$

Therefore, $\langle G\rangle = G_o \exp\left(\langle G_2/G_o\rangle\right)$

We can show that

$$\langle G_2/G_o\rangle = \frac{k^2}{G(\bar{r})}\int\int G(\bar{r},\bar{r}_1)\langle\epsilon_1(\bar{r}_1)\epsilon_1(\bar{r}_2)\rangle G(\bar{r}_1,\bar{r}_2)G(\bar{r}_2)\,dV_1\,dV_2$$

$$= -2\pi^2k^2\int_0^L dz\int_0^\infty \Phi_n(\kappa)\,\kappa\,d\kappa = -\int_0^L \alpha(z)dz$$

$$\langle\epsilon_1(\bar{r}_1)\epsilon_1(\bar{r}_2)\rangle = 4\langle n_1(r_1)n_1(r_2)\rangle = \delta(z_1-z_2)A(\bar{\rho}_1-\bar{\rho}_2)$$

$$A(\rho) = 16\pi^2\int_0^\infty J_o(\kappa\rho)\Phi_n(\kappa)\,d\kappa$$

$\Phi_n(\kappa)$ is the spectrum of the refractive index fluctuation

- Consistent with moment equation solution

Two-frequency MCF - derivation

$$\Gamma(\bar{r}_1,\omega_1;\bar{r}_2,\omega_2) = \langle GG^* \rangle = \Big\langle G(\bar{r}_1,\omega_1)G^*(\bar{r}_2,\omega_2)\Big\rangle$$

- Phase perturbation series up to ϵ^2

$$\Gamma = G_o(\bar{r}_1,\omega_1)G_o^*(\bar{r}_2,\omega_2)\Big\langle \exp\left(\psi_1+\psi_2+\psi_1^*+\psi_2^*+\mathcal{O}(\epsilon^3)\right)\Big\rangle$$

$$\psi_1=\psi_1(\bar{r}_1,\omega_1),\ \psi_2=\psi_2(\bar{r}_1,\omega_1),\ \psi_1^*=\psi_1^*(\bar{r}_2,\omega_2),\ \psi_2^*=\psi_2^*(\bar{r}_2,\omega_2)$$

- Assume normal distribution

$$\Big\langle \exp\left(\psi_1+\psi_2+\psi_1^*+\psi_2^*\right)\Big\rangle = \exp\left(\langle\psi_2\rangle+\langle\psi_2^*\rangle+\frac{1}{2}\langle\psi_1^2\rangle+\frac{1}{2}\langle\psi_1^{2*}\rangle+\langle\psi_1\psi_1^*\rangle+\mathcal{O}(\epsilon^3)\right)$$

$$= \exp\left[\langle G_2/G_o\rangle+\langle (G_2/G_o)^*\rangle+\langle\psi_1\psi_1^*\rangle+\mathcal{O}(\epsilon^3)\right]$$

$$\frac{G_2}{G_o}=\frac{G_2(\bar{r}_1,\omega)}{G_o(\bar{r}_1,\omega)},\ \left(\frac{G_2}{G_o}\right)^*=\frac{G_2^*(\bar{r}_2,\omega)}{G_o^*(\bar{r}_2,\omega)},\ \psi_1=\psi_1(\bar{r}_1,\omega_1),\ \psi_1^*=\psi_1^*(\bar{r}_2,\omega_2)$$

- This requires

$$\langle\psi_1\psi_1^*\rangle = \Big\langle \left[\chi_1(\bar{r}_1,\omega_1)+iS_1(\bar{r}_1,\omega_1)\right]\left[\chi_1(\bar{r}_2,\omega_2)+iS_1(\bar{r}_2,\omega_2)\right]^* \Big\rangle = B_\chi+B_S+i\left[B_{S\chi}-B_{\chi S}\right]$$

$$B_\chi=\langle\chi_1(\bar{r}_1,\omega_1)\chi_1(\bar{r}_2,\omega_2)\rangle,\ B_S=\langle S_1(\bar{r}_1,\omega_1)S_1(\bar{r}_2,\omega_2)\rangle,$$

$$B_{S\chi}=\langle S_1(\bar{r}_1,\omega_1)\chi_1(\bar{r}_2,\omega_2)\rangle,\ B_{\chi S}=\langle\chi_1(\bar{r}_1,\omega_1)S_1(\bar{r}_2,\omega_2)\rangle$$

Final expression for two-frequency MCF

- Correlation B_χ is given by

$$B_\chi = 2\pi^2 k_1 k_2 \int_0^L dz \int_0^\infty \kappa \, d\kappa \, g_\chi(\kappa, z) J_o(\kappa P) \Phi_n(\kappa, z); \; P = |\rho_d z / L|$$

$$g_\chi = \Re[h_1 h_2^* - h_1 h_2], \quad g_S = \Re[h_1 h_2^* + h_1 h_2]$$

$$g_{S\chi} = -\Im[h_2 h_1^* + h_2 h_1], \quad g_{\chi S} = -\Im[h_1 h_2^* - h_1 h_2]$$

where $h_1 = \exp\left(-i \frac{z}{L} \frac{(L-z)}{2k_1} \kappa^2\right), \quad h_2 = \exp\left(-i \frac{z}{L} \frac{(L-z)}{2k_2} \kappa^2\right)$

- Final expression

$$\Gamma(\bar{r}_1, \omega_1; \bar{r}_2, \omega_2) = G_o(\bar{r}_1, \omega_1) G_o^*(\bar{r}_2, \omega_2) \exp(-H)$$

$$H = 4\pi^2 \int_0^L dz \int_0^\infty \kappa \, d\kappa \left[\frac{k_1^2 + k_2^2}{2} - k_1 k_2 \, g \, J_o(\kappa P) \right] \Phi_n(\kappa, z)$$

Two-frequency MCF for discrete scatterers

- For discrete scatterers

$$2\pi k^2\Phi_n(\kappa) = \frac{b}{4\pi}p(s), \quad \kappa = ks \;\rightarrow$$

$$\Gamma(\bar{r}_1,\omega_1;\bar{r}_2,\omega_2) = G_o(\bar{r}_1,\omega_1)G_o^*(\bar{r}_2,\omega_2)\exp(-H)$$

$$H = \tau_a + \int_0^L dz\,\frac{b}{2}\int_0^\infty s\,ds\,[1 - gJ_o(ksP)]\,p(s)$$

where

$$g = \exp\left[+i\frac{z(L-z)}{2L}k_d s^2\right], \quad \tau_a = \int_0^L a\,dz = \text{absorption depth}$$

- Assume Gaussian phase function

$$p(s) = 4\alpha_p \exp\left(-\alpha_p s^2\right) \longrightarrow H = \tau_a + \int_0^L dz\,b\left[1 - \frac{\exp\left(-\frac{k^2P}{4(\alpha_p - A)}\right)}{\left(1 - \frac{A}{\alpha_p}\right)}\right]$$

where

$$A = \frac{iz(L-z)}{2L}k_d \quad B = (1 - A/\alpha_p)^{-1}$$

- Approximation

$$\exp(-H) = \exp(-\tau_o) + F_s\exp\left(-\tau_a - \int_0^L dz\,b\left[(1-B) - \frac{Bk^2P^2}{F_s 4(\alpha_p - A)}\right]\right)$$

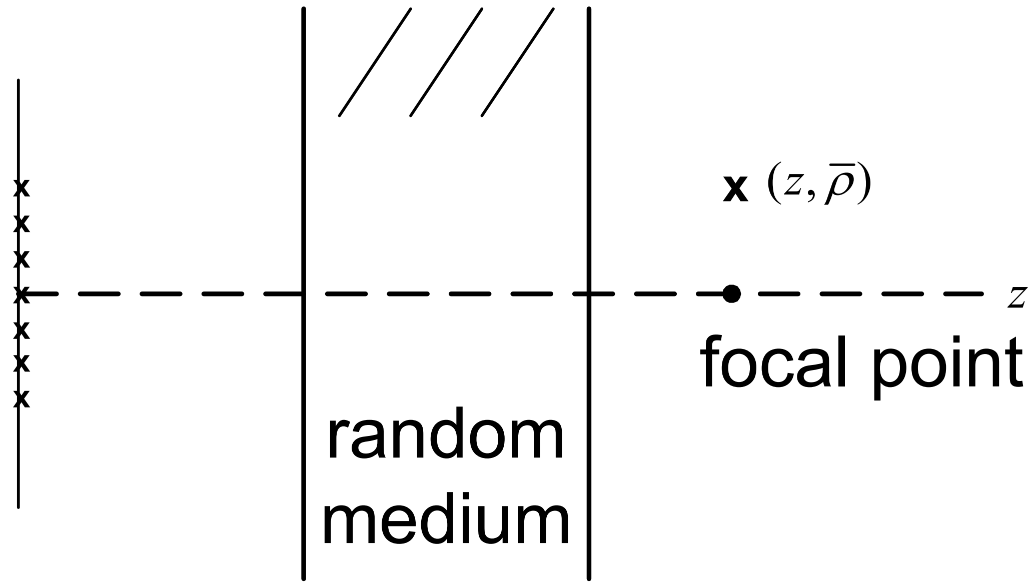
$$F_s = 1 - \exp\left(-\int dz\,b\,B\right)$$

Comparison with known plane wave solution

- Plane wave solution

$$\Gamma = \frac{1}{\cos\left(i4\alpha\right)^{1/2}} \exp\left[-\frac{\tan(i4\alpha)^{1/2}}{(i4\alpha)^{1/2}} \rho_d^2\right]$$

4. General solution for imaging



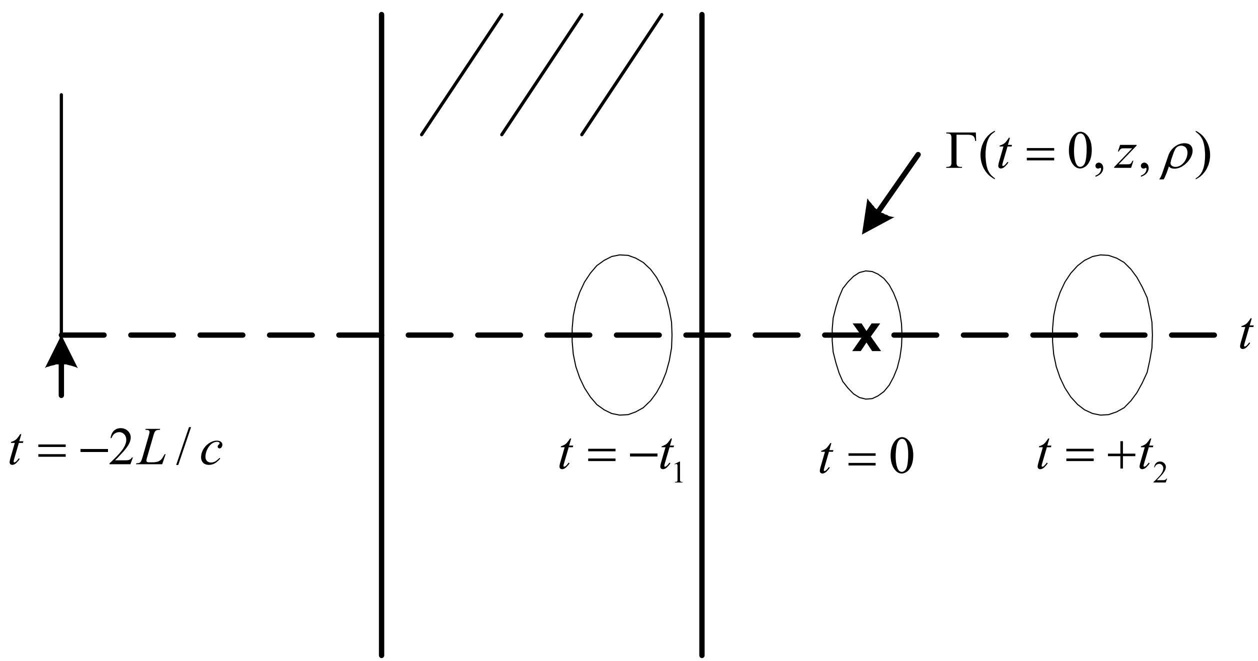
$$\Gamma(t, z, \bar{\rho}) = \frac{1}{\pi \Delta\omega} \sqrt{\frac{\pi}{2}} \int d\omega_d \, \exp(-i\omega_d t) \Gamma(\omega_d) \exp\left(-\frac{\omega_d^2}{2\Delta\omega^2}\right)$$

$$\Gamma(\omega_d) = \sum_p \sum_{p'} \sum_q \sum_{q'} \langle g_1 \, g_2 \, g_1'^* \, g_2'^* \rangle$$

General solution for imaging

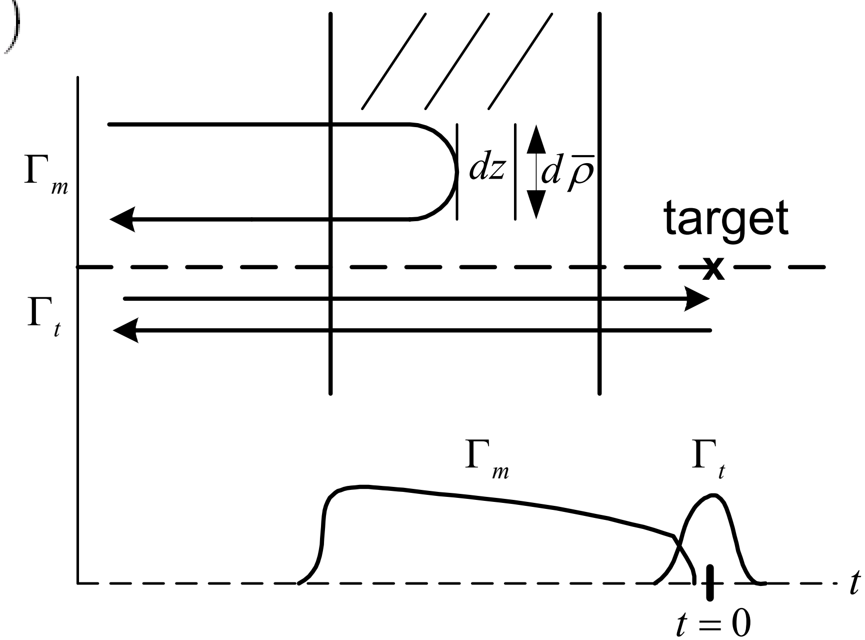
(1) Pulse $\Gamma(t, z, \rho)$ at $t = 0$ and $t \neq 0$

- Origin $t = 0$ is at $(z = L, \rho = 0)$



General solution for imaging

(2) Backscattered pulse $\Gamma(t)$



$$\Gamma = \Gamma_m(t)\, b\, p(s=2) + \Gamma_t(t)\sigma_b$$

$$\Gamma_m(t) = \int d\bar{\rho} \int dz\, \Gamma(t, z, \bar{\rho})$$

$$= \text{medium scattering}$$

$$\Gamma_t(t) = \Gamma(t, z = L, \rho = 0)$$

$$= \text{target scattering}$$

$$p(s) = \text{Henyey Greenstein} = \frac{1+g}{(1-g)^2}\left[1 + \left(\frac{s}{s_o}\right)^2\right]^{-3/2}$$

$$s = 2\sin\frac{\theta}{2}, \quad s_o^2 = \frac{(1-g)^2}{g}, \quad s = 2(\text{back})$$

$$\sigma_b = \text{backscattering crosssection} = A\lambda^2$$

5. Approximate analytical solution

$$\Gamma(\omega_d) = \sum_{p} \sum_{p'} \sum_{q} \sum_{q'} \langle g_1(\omega_1)\, g_2(\omega_1)\, g_1'^{*}(\omega_2)\, g_2'^{*}(\omega_2) \rangle$$

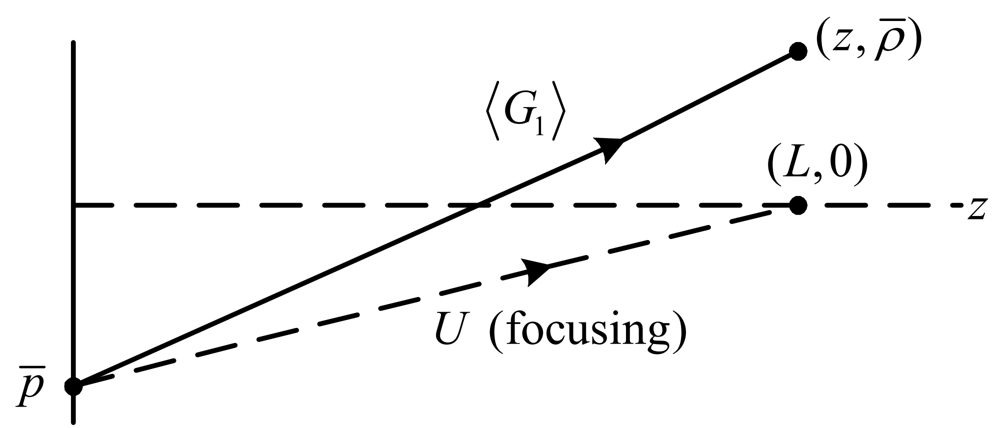
- Coherent field $\langle g_1(\omega) \rangle$ etc.
- Two-frequency MCF $\langle g_1(\omega_1)\, g_1'^{*}(\omega_2) \rangle$ etc.

General formulation – Parabolic approximation coherent Green's function

$$\langle g_1 \rangle = \langle G_1(\bar{\rho}, \bar{r}_o, \omega_1) \rangle u(\bar{p}, \bar{r}_f, \omega_1)$$

$$\langle G_1 \rangle = G_o \exp\left(-\int \alpha \, dz\right) = \frac{1}{4\pi z} \exp\left(+ik_1 z + i\frac{k_1}{2z}|\bar{\rho} - \bar{p}|^2 - \frac{\tau_o}{2}\right)$$

$$u = \exp\left(-ik_1 L - i\frac{k_1 p^2}{2L}\right), \quad \bar{p} = x\hat{x} + y\hat{y}$$



$$\tau_o = \int dz(a+b); \text{ a=absoption, b=scattering coefficient}=2\alpha(z)$$

Coherent field

$$\langle g_1 \rangle = \langle G_1 \rangle u = I_p(k_1) \exp\left(-\tau_o/2\right)$$

$$I_p(k_1) = \frac{1}{4\pi z} \exp\left[ik_1(z-L) + \frac{ik_1}{2z}\rho^2 + \frac{ik_1 p^2}{2}\left(\frac{1}{z} - \frac{1}{L}\right) - i\frac{k_1}{z}\bar{p}\cdot\bar{\rho}\right]$$

$$\Gamma = \sum_{i=1}^{7} \sum_{p} \sum_{p'} \sum_{q} \sum_{q'} I_i$$

$$I_1 = \langle g_1 \rangle \langle g_2 \rangle \langle g_1'^* \rangle \langle g_2'^* \rangle = I_p(k_1) I_q(k_1) I_{p'}(k_2) I_{q'}(k_2) \exp(-2\tau_o)$$

$$I_2 = \langle g_1 \rangle \langle g_1'^* \rangle \langle g_{2f} g_{2f}'^* \rangle$$

$$\langle g_1 \rangle = I_p(k_1) \exp(-\tau_o/2)$$

$$\langle g_1'^* \rangle = I_{p'}(k_2) \exp(-\tau_o/2)$$

$$\langle g_{2f} g_{2f}'^* \rangle = I_q(k_1) I_q(k_2) F_s \exp\left[-E_f - D|\bar{q} - \bar{q}'|^2\right]$$

$$F_s = 1 - \exp\left[\int dz\, b\, B_p\right] \qquad E_f = \int dz\, b\, (1 - B_p)$$

$$D = \int dz\, b\, B_p \frac{k^2}{4\,(\alpha_p - A_p)} \frac{(z_m - z)}{z_m}$$

$$A_p = \frac{iz\,(z_m - z)}{2z_m} k_d; \quad B_p = \left(1 - \frac{A_p}{\alpha_p}\right)^{-1}$$

General formulation – Focused array and aperture – Poison's sum formula

$$\sum_p \langle g_1 \rangle = \sum_p \langle G_1 \rangle u$$

- Linear array

$$\sum_{p=-N}^{N} f(p) \cong \frac{(2N+1)}{2a} \int_{-a}^{a} f(x)\,dx \cong \frac{(2N+1)}{2a} \int_{-\infty}^{\infty} \frac{2}{\sqrt{\pi}} \exp\left(-\frac{x^2}{a^2}\right) f(x)\,dx$$

- Square array

$$\sum_p = \sum_{p_x} \sum_{p_y} = \left(\frac{(2N+1)}{2a}\right)^2 \int\int \frac{4}{\pi} \exp\left(-\frac{x^2+y^2}{a^2}\right) f(x,y)\,dx\,dy$$

$$I(A,B,C,\alpha,\beta) = \int\int dx\,dy\ \exp\left(-Ax^2 - By^2 + Cxy + \alpha x + \beta y\right)$$

$$= \frac{\pi}{\sqrt{AB - C^2/4}} \exp\left[\frac{\alpha^2 B + \beta^2 A + \alpha\beta C}{4(AB - C^2/4)}\right]$$

$$\sum_p \sum_{p'} \sum_q \sum_{q'} \langle g_1 g_2 g_1'^* g_2'^* \rangle$$

$$\begin{aligned}
= \sum\sum\sum\sum \quad & \langle g_1 \rangle \langle g_2 \rangle \langle g_1'^* \rangle \langle g_2'^* \rangle \to I_o^2 \\
& + \langle g_1 \rangle \langle g_1'^* \rangle \langle g_{2f} g_{2f}'^* \rangle \to I_o I_f \\
& + \langle g_2 \rangle \langle g_2'^* \rangle \langle g_{1f} g_{1f}'^* \rangle \to I_o I_f \\
& + \langle g_{1f} g_{1f}'^* \rangle \langle g_{2f} g_{2f}'^* \rangle \to I_f^2 \\
& + \langle g_1 \rangle \langle g_2'^* \rangle \langle g_{2f} g_{1f}'^* \rangle \to I_o I_f (*) \\
& + \langle g_2 \rangle \langle g_1'^* \rangle \langle g_{1f} g_{2f}'^* \rangle \to I_o I_f (*) \\
& + \langle g_{1f} g_{2f}'^* \rangle \langle g_{2f} g_{1f}'^* \rangle \to I_f^2 (*)
\end{aligned}$$

$$= I_o^2 + 4 I_o I_f + 2 I_f^2$$

$(*)$ = backscattering enhancement (cross term)

6. Final expression for $\Gamma(t, z, \bar{\rho})$

$$\Gamma = \Gamma_{\text{target}}\sigma_b + \Gamma_{\text{medium}}[b\,p(s=2)]$$

$$\Gamma_{\text{target}}\left(z_m, \bar{\rho}_m, t\right) = Const \int \Gamma_o\left(z_m, \bar{\rho}_m, \omega_d\right) \exp\left(-2\tau_{am}\right) \exp\left(\Phi\right)\, d\omega_d$$

$$\tau_{am} = \int_{d_1}^{z_m} a_b dz = a_b(z_m - d_1) \qquad \bar{\rho}_m = \rho_{mx}\hat{x} + \rho_{my}\hat{y}$$

$$\Phi = i\frac{\omega_d}{2cz_m}2\left(\rho_{mx}^2 + \rho_{my}^2\right) + i\frac{\omega_d}{c}2\left(z_m - L\right) - \frac{\omega_d^2}{2\left(\Delta\omega\right)^2} - i\omega_d t$$

$$\Gamma_o(z_m, \rho_m, \omega_d) = \left(I_o^2 + 4I_oI_f + 2I_f^2\right)\frac{1}{z_m^4}$$

$$\text{Without enhancement } = \left(I_o^2 + 2I_oI_f + I_f^2\right)\frac{1}{z_m^4}$$

$$I_o(A,B,C,\alpha,\beta) = \Theta_x \Theta_y \exp(-\tau_{sm})$$

$$\Theta_x = \frac{\pi}{\sqrt{AB - C^2/4}} \exp\left[\frac{\alpha_x^2 B + \beta_x^2 A + \alpha_x \beta_x C}{4(AB - C^2/4)}\right]$$

$$\Theta_y = \frac{\pi}{\sqrt{AB - C^2/4}} \exp\left[\frac{\alpha_y^2 B + \beta_y^2 A + \alpha_y \beta_y C}{4(AB - C^2/4)}\right]$$

$$\tau_{sm} = \int_{d_1}^{z_m} b dz$$

$$A = \frac{1}{a^2} - i\frac{k_1}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right),\ B = \frac{1}{a^2} + i\frac{k_2}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right),\ C = 0$$

$$\alpha_x = -i\frac{k_1 \rho_{mx}}{z_m}, \beta_x = i\frac{k_2 \rho_{mx}}{z_m} \qquad \alpha_y = -i\frac{k_1 \rho_{my}}{z_m}, \beta_x = i\frac{k_2 \rho_{my}}{z_m}$$

$$I_f\left(A_f, B_f, C_f, \alpha_f, \beta_f\right) = F_s \Upsilon_x \Upsilon_y \exp\left(-E_f\right)$$

$$\Upsilon_x = \frac{\pi}{\sqrt{A_f B_f - C_f^2/4}} \exp\left[\frac{\alpha_{fx}^2 B_f + \beta_{fx}^2 A_f + \alpha_{fx}\beta_{fx} C_f}{4\left(A_f B_f - C_f^2/4\right)}\right]$$

$$\Upsilon_y = \frac{\pi}{\sqrt{A_f B_f - C_f^2/4}} \exp\left[\frac{\alpha_{fy}^2 B_f + \beta_{fy}^2 A_f + \alpha_{fy}\beta_{fy} C_f}{4\left(A_f B_f - C_f^2/4\right)}\right]$$

$$A_f = \frac{1}{a^2} + \frac{D}{F_s} - i\frac{k_1}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), B_f = \frac{1}{a^2} + \frac{D}{F_s} + i\frac{k_2}{2}\left(\frac{1}{z_m} - \frac{1}{L}\right), C_f = 2\frac{D}{F_s}$$

$$F_s = 1 - \exp\left(-\int_{d_1}^{z_m} dzb\ B_p\right) \qquad D = \int_{d_1}^{z_m} dzb\ B_p \frac{k^2}{4\left(\alpha_p - A_p\right)} \frac{\left(z_m - z\right)}{z_m}$$

$$A_p = \frac{iz\left(z_m - z\right)}{2z_m} k_d \quad E_f = \int_{d_1}^{z_m} dzb\left(1 - B_p\right) \qquad B_p = \left(1 - \frac{A_p}{\alpha_p}\right)^{-1}$$

$$\alpha_{fy} = -i\frac{k_1 \rho_{my}}{z_m}, \beta_{fx} = i\frac{k_2 \rho_{my}}{z_m} \quad \alpha_{fx} = -i\frac{k_1 \rho_{mx}}{z_m}, \beta_{fx} = i\frac{k_2 \rho_{mx}}{z_m}$$

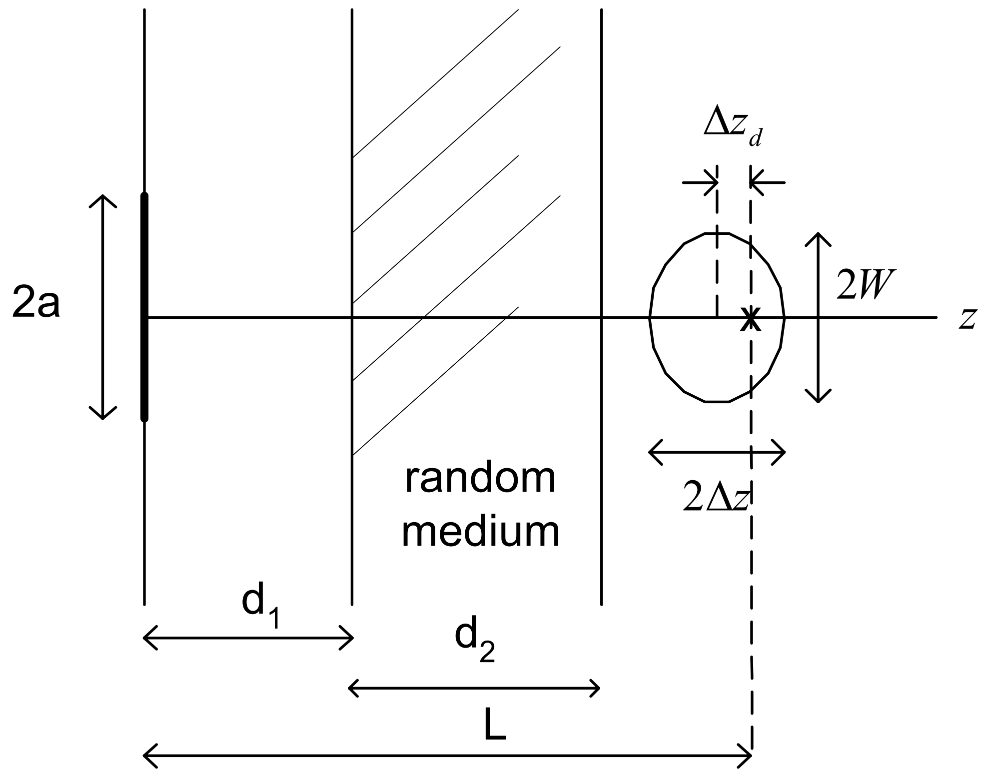
Transverse Resolution

$$\frac{W^2}{W_o^2} = 1 + \frac{a^2}{\rho_{coh}^2}, \qquad \rho_{coh}^2 = \frac{4\alpha_p}{k^2\tau_s}$$

$$W_o = \text{ free space}$$

$$\rho_{coh}^2 = \frac{4\alpha_p}{k^2\tau_s}, \; \rho_{coh} = \text{ coherence length}$$

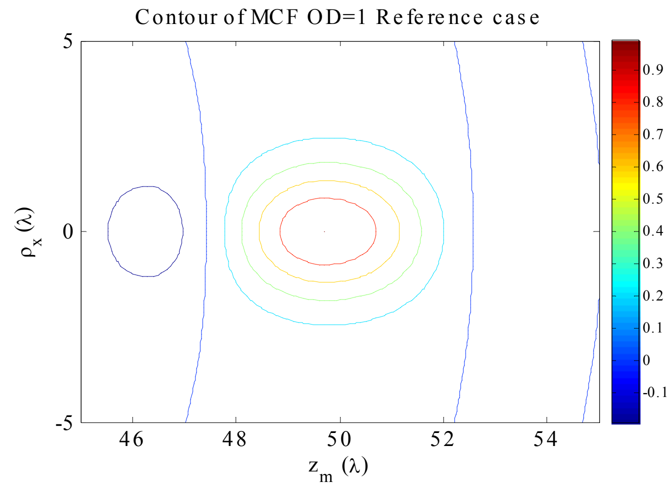
Numerical Examples



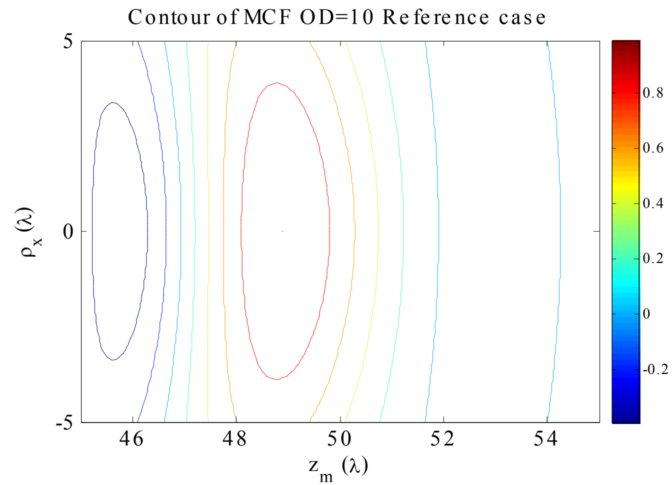
$\Delta z =$ pluse spread
$W =$ lateral resolution
$\Delta z_d =$ delay due to multiple scattering
$a = 5\lambda$
$\Delta\omega/\omega_o = 0.2$
$g = 0.85$
$W_o = 0.9$
$L = 50\lambda$
$d_1 = d_2 = 25\lambda$
$\sigma_b = A\lambda^2, A = 1$

Pulse spread, delay and lateral resolution

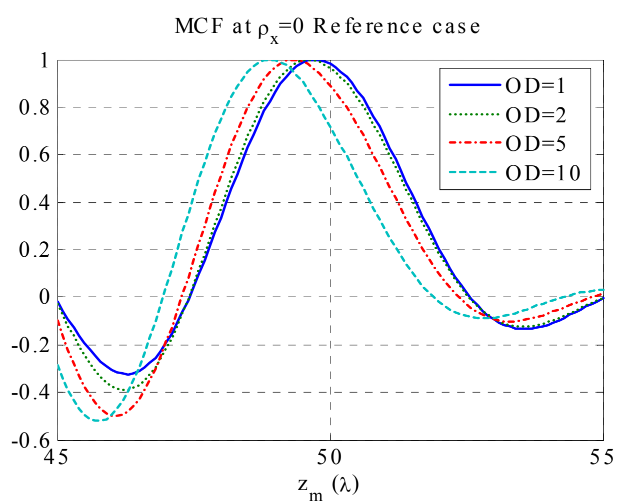
OD = 1



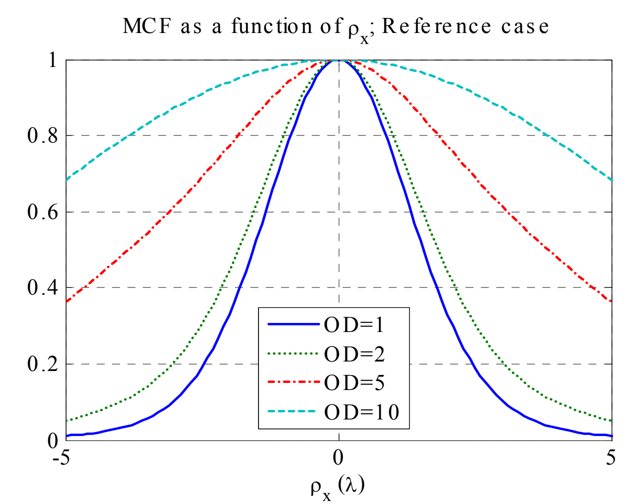
OD = 10



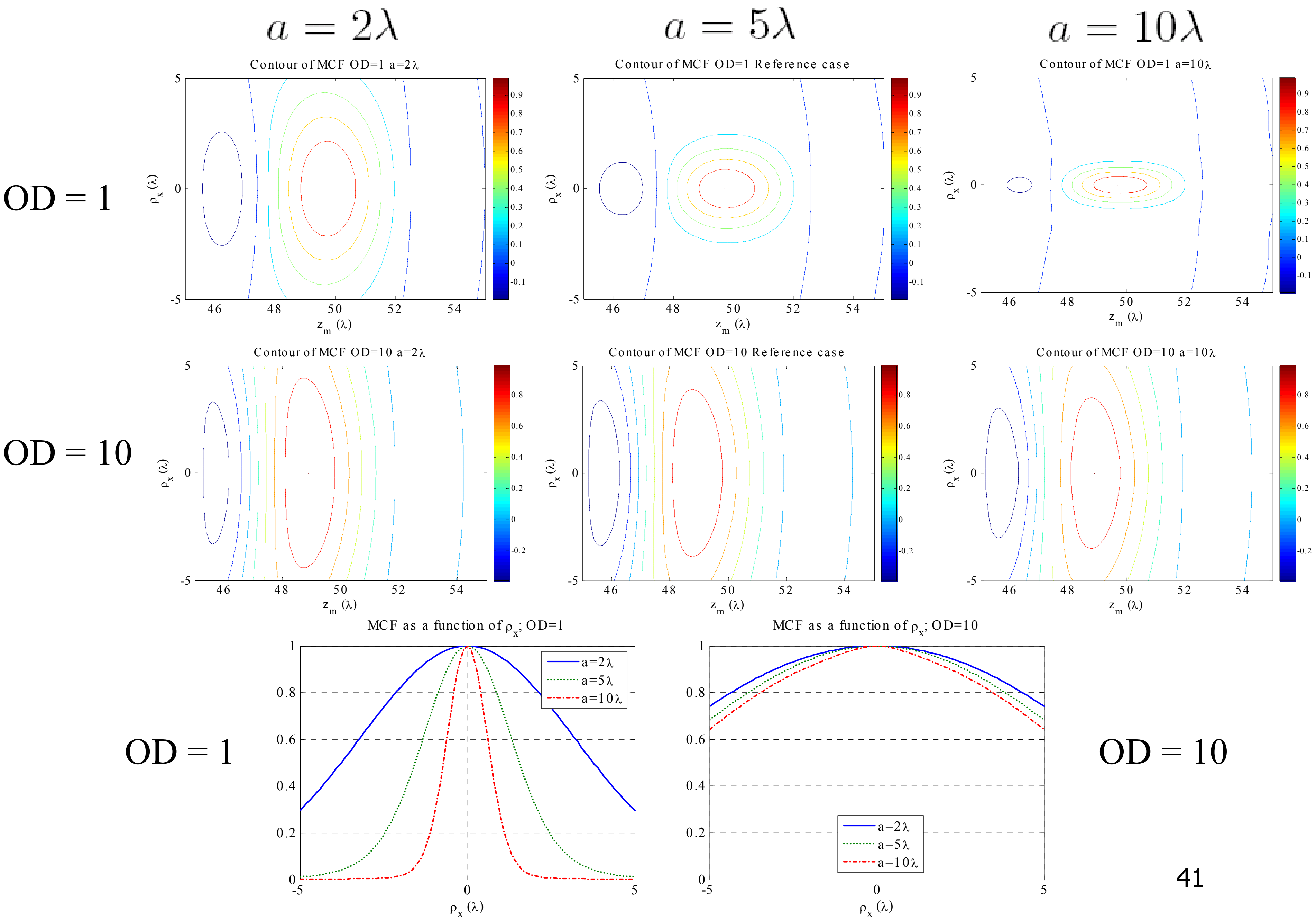
Pulse spread and delay



Lateral resolution



Effect of aperture size

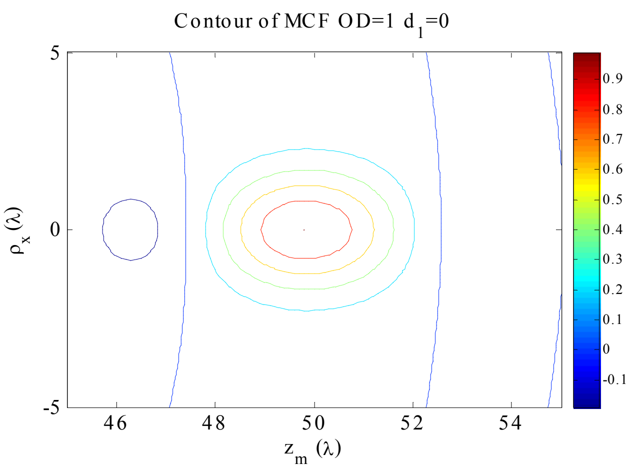


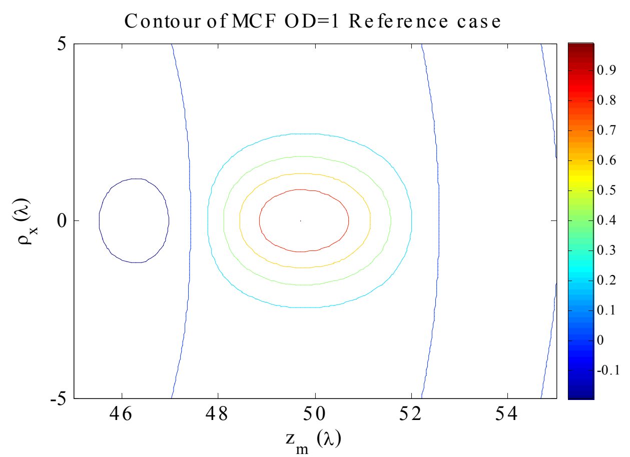
Shower curtain effects

- Resolution is better if $d_1 = 25\lambda$

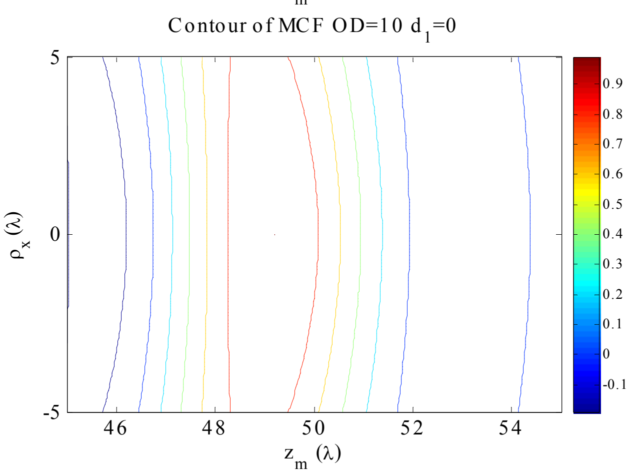
$d_1 = 0$ $\qquad\qquad$ $d_1 = 25\lambda$

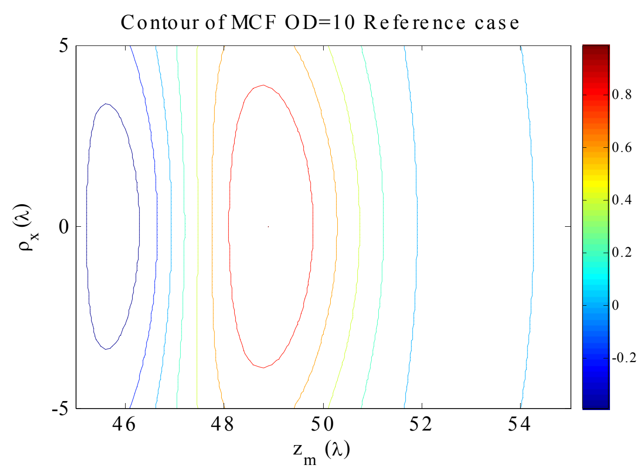
OD = 1



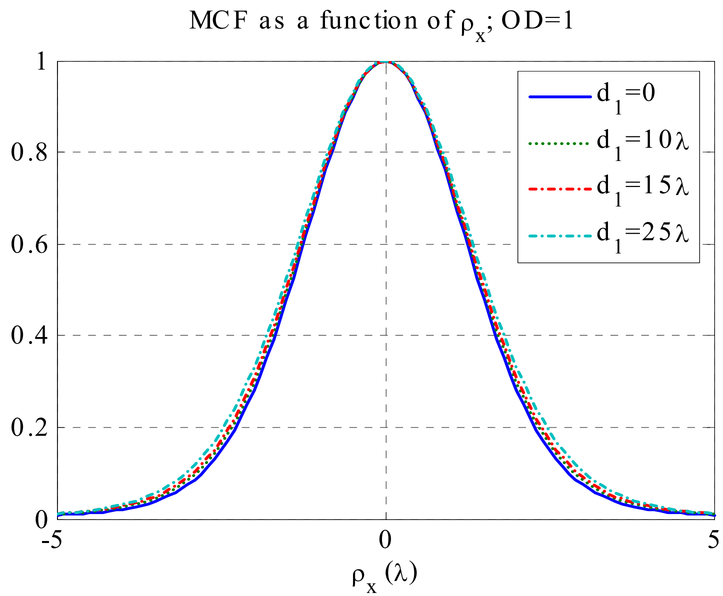


OD = 10

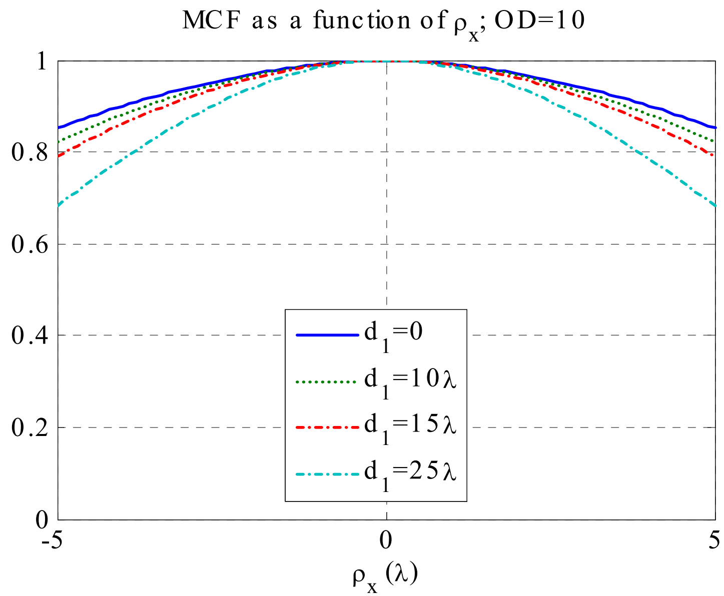




Effects of medium thickness on transverse resolution

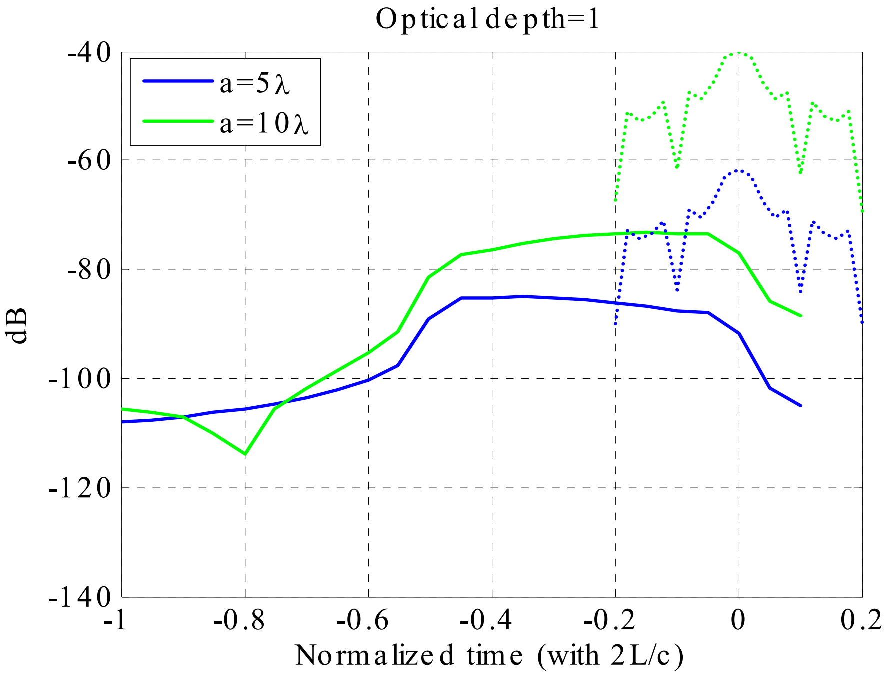


OD = 1

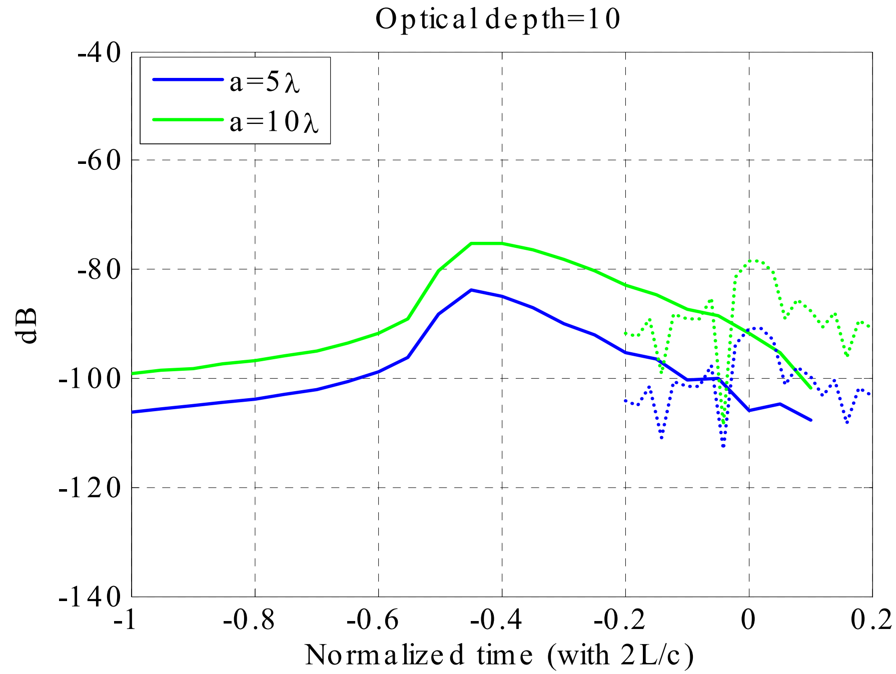


OD = 10

Effect of aperture size

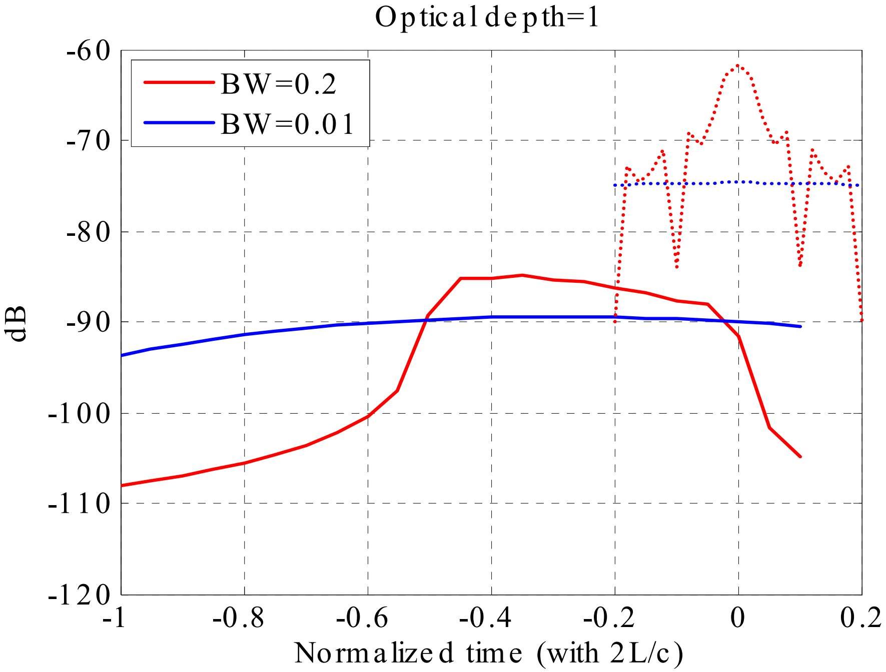


OD = 1

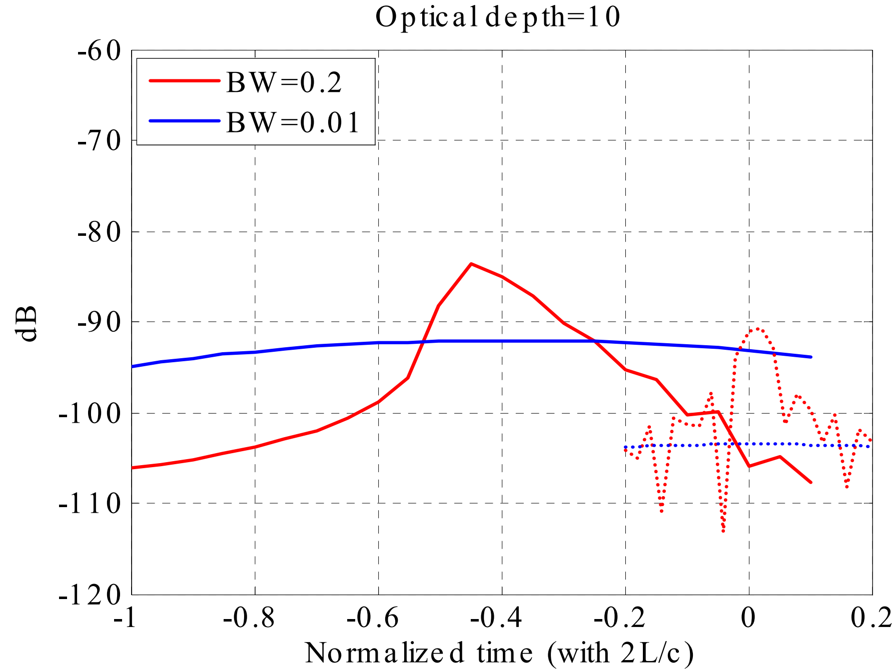


OD = 10

Effect of bandwidth

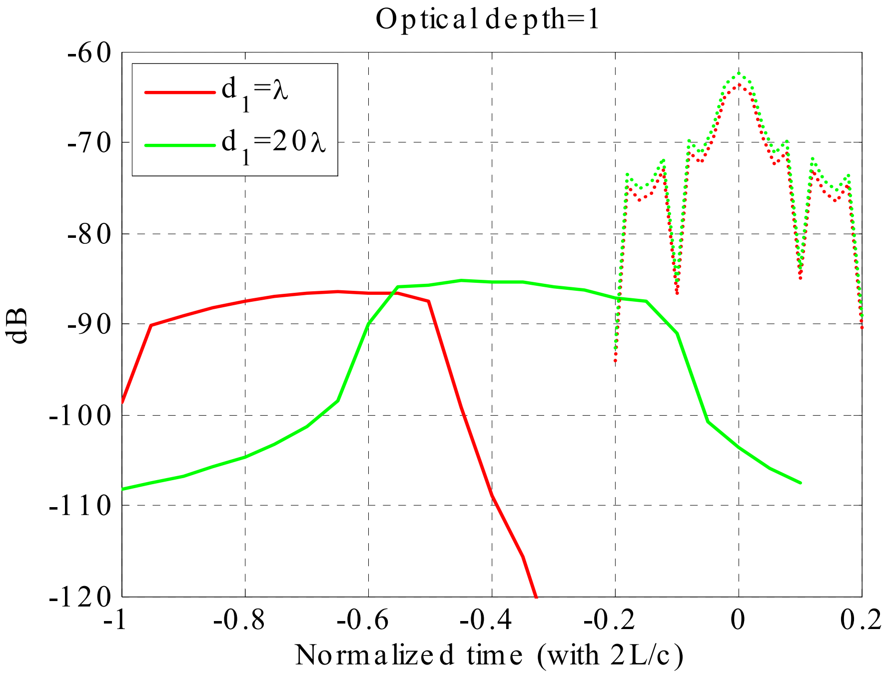


OD = 1

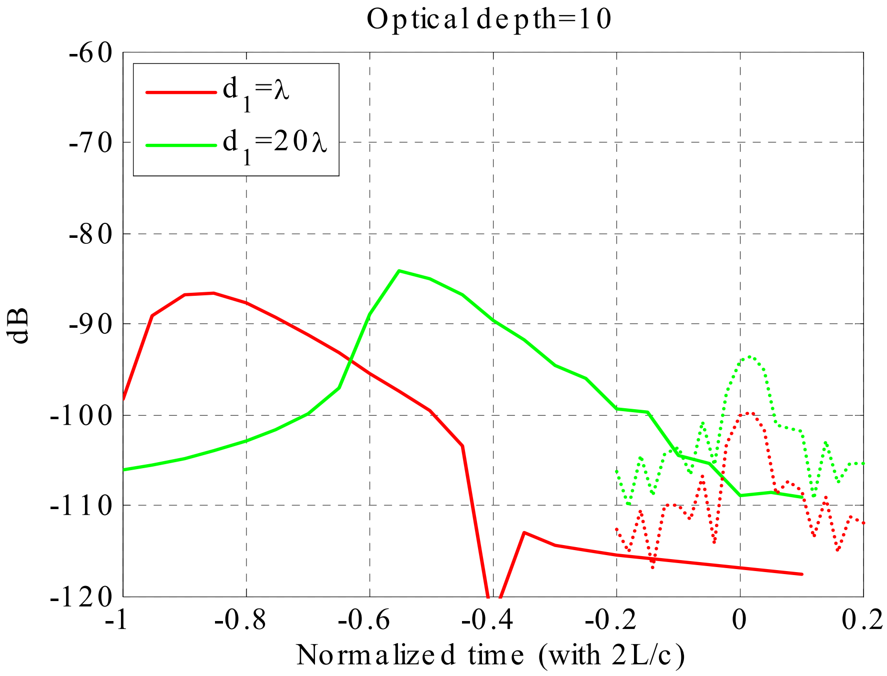


OD = 10

Effect of d_1

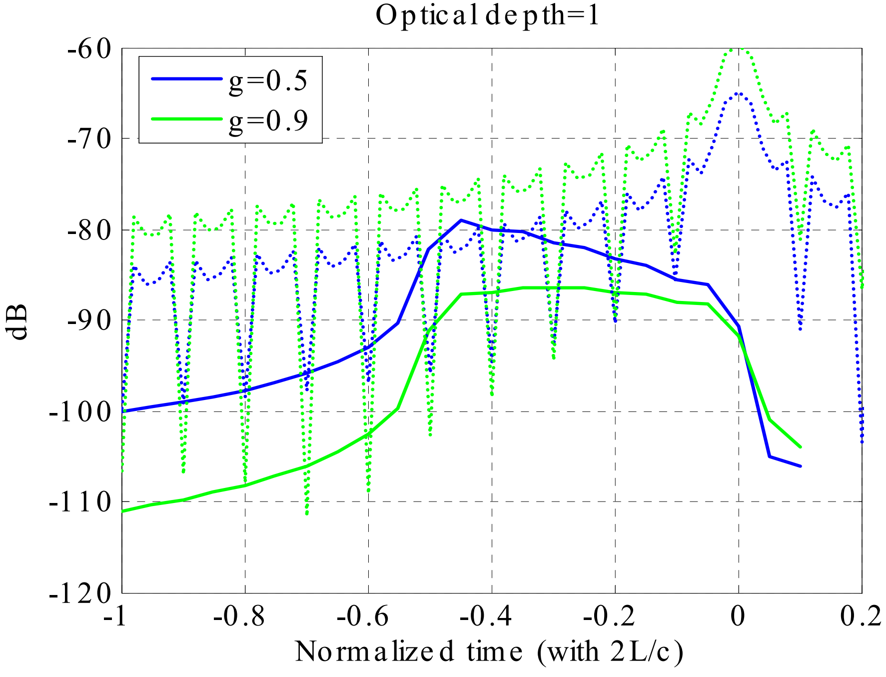


OD = 1

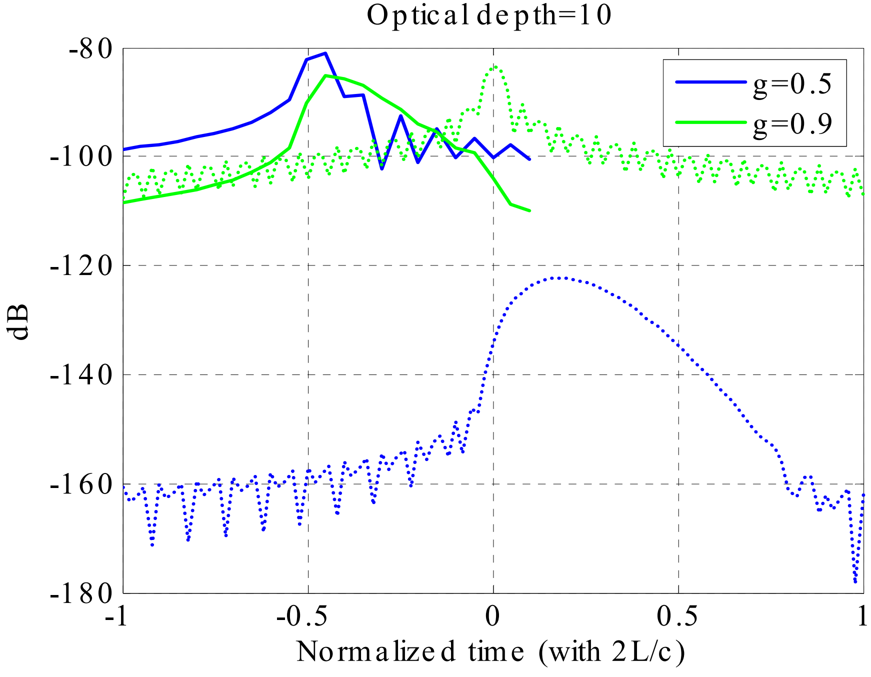


OD = 10

Effect of g (anisotropy factor)

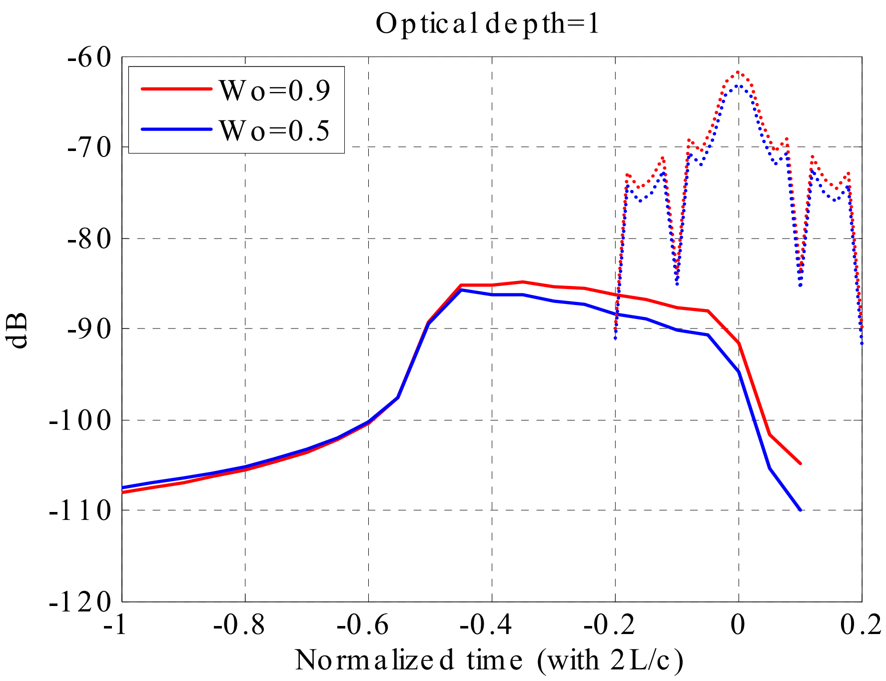


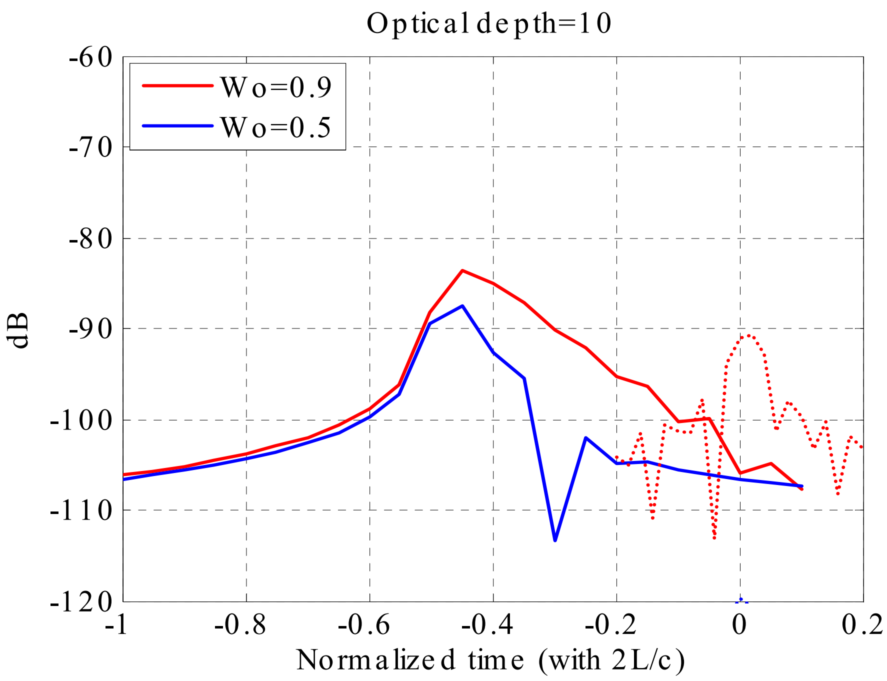
OD = 1



OD = 10

Effect of albedo





OD = 1

OD = 10

Conclusions

- Detection of objects behind random medium
- Use of array of transmitters and receivers
  - Focused
  - UWB pulse
- Two frequency MCF
- Transverse and longitudinal resolution
- Coherence length, shower curtain effects, backscattering enhancement
- Relationship with OCT (Optical coherence tomography), SAR, Confocal imaging

Future problems

- Point objects (this paper)
- Large objects (Kirchhoff)
- Objects with different size and shape, conducting and dielectric, polarimetric
- Random medium – layer (this paper)
- Rough surfaces, terrain, ocean surfaces, bottom buried objects, void, cavity, waveguides