Snow thickness estimation using correlation functions at C-band

Sermsak Jaruwatanadilok, Yasuo Kuga, and Akira Ishimaru
Department of Electrical Engineering, Box 352500
University of Washington
Seattle, WA, USA, 98195
sermsak@ee.washington.edu

Ziad A. Hussein, Kyle C. McDonald
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, CA 91109

***Abstract*— An accurate measurement of snow-layer thickness on the ground is a critical process for estimating the equivalent water content of snow. Previous studies were mainly based on the analysis of the backscattering cross-section of two co-polarized signals at different frequencies. The snow thickness was inferred by comparison with the analytical model. Recently, we have applied the angular and frequency correlation functions (ACF/FCF) for the estimation of sea-ice thickness. One of the advantages of the ACF/FCF method is suppression of the interfering volume scattering, which results in better accuracy and reliability of thickness estimation. We apply a simplified 1-D three-layer model for the analysis. The layers are air, snow, and ground. The interfaces between layers are modeled as rough surfaces. Within the layers, there are small inclusions which introduce the volume scattering. Rough surfaces are modeled by the Kirchhoff approximation methods. The volume scattering is calculated using the quasi-crystalline approximation with coherent potential approximation (QCA-CP) for small particles. The ACF/FCF works by correlating two signals with different frequencies and/or incident angles. Using this model, we can determine the behavior of ACF/FCF and use it for snow thickness retrieval.**

Keywords- correlation function, snow thickness, snow water content, rough surface scattering, volume scattering

I. INTRODUCTION

Snow Water Equivalent (SWE) is a crucial parameter in hydrological studies since it dictates the amount of water available [1]-[2]. The snow depth is a very important parameter in determining the SWE. Therefore, there exists the need for methods to accurately estimate the snow thickness. Most previous works on snow thickness estimation are based on analysis of the backscattering cross-section of polarized signals at different frequencies [3]-[9]. The surface scattering and volume scattering are normally considered separately. Due to the dense properties of the snow, the dense media radiative transfer is usually used to model the volume scattering. The surface scatterings are from rough interfaces between air and snow or snow and ground. Several methods are applied for surface scattering modeling, including the small perturbation model, Kirchhoff approximation, and integral equation method (IEM). Each method has a certain range of validity due to the surface characteristics and the wavelength.

Here, we present a different method based on correlation function. The characteristics of the correlation of two waves with different frequencies and different incident angles are investigated. We show that some of the features of the correlation function are useful in snow parameter estimation, including the snow depth and snow density. This method provides additional information to better estimate the snow depth.

II. MODEL

We model the snow on the ground as a three-layered medium which consists of air, snow, and ground as shown in Fig. 1. The interface of each layer is assumed to be a rough surface. The electrical properties of each layer and the surface properties of each interface in our calculations are given in Table I.

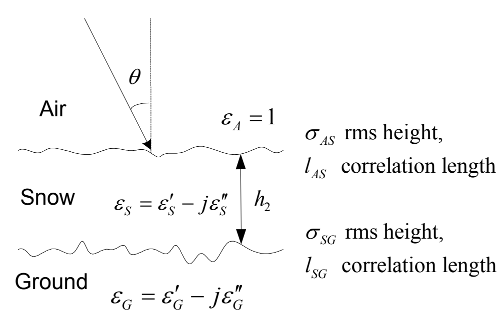

Figure 1. One-dimensional three-layered medium model

TABLE I. MEDIUM PROPERTIES IN THE CALCULATIONS

	Dielectric constant	*Rms height (cm)*	*Correlation length (cm)*
Snow	See TABLE II	$\sigma_{AS} = 0.5$	$l_{AS} = 8.0$
Ground	5.0	$\sigma_{SG} = 1.2$	$l_{SG} = 12.0$

The dielectric properties of snow are quite complex depending on several parameters such as frequency, temperature, snow density, and water content. In this paper, we only consider dry snow. We employ the model explained by

Hallikainen [10] and Ulaby [11]. The snow layer is modeled as air background with small spherical ice particles. The effective dielectric constant of the snow bulk can be calculated using the Quasi-crystalline Approximation with Coherent Potential (QCA-CP) mixing formula:

$$\varepsilon_{eff} = \varepsilon_1 + \frac{3\varepsilon_{eff} f \dfrac{\varepsilon_2-\varepsilon_1}{3\varepsilon_{eff}+\varepsilon_2-\varepsilon_1}}{1-f\dfrac{\varepsilon_2-\varepsilon_1}{3\varepsilon_{eff}+\varepsilon_2-\varepsilon_1}}$$

where f is the fractional volume, ε_1 is the dielectric constant of the host (air), and ε_2 is dielectric constant of the inclusions (ice). Density (D) is related to fractional volume (f) by $f = D/0.96$. Table II shows the density, grain size (diameter) of ice particles and the corresponding bulk dielectric constant of snow for three classes of snow.

TABLE II. SNOW DIELECTRIC CONSTANTS

Snow class	I	II	III
Density (g cm^{-3})	0.220	0.323	0.400
Grain size (mm)	4.0	2.6	2.0
Bulk dielectric constant	1.3695 - 1.6053e-5j	1.5866 - 2.7763e-5j	1.7641 - 3.7912e-5j

From these snow parameters, we calculate the correlation function. In this simple model, we assume that the volume and surface scattering are independent. The contributions of correlation function from surface scattering are based on the Kirchhoff approximation, while the contributions from volume scattering are calculated based on the distorted Born approximation where the coherent component is based on QCA-CP [12].

III. SURFACE SCATTERING – KIRCHHOFF APPROXIMATION

For C-band, the given surface roughness and correlation length falls into the Kirchhoff approximation range of validity. The formulation for the ACF/FCF for Kirchhoff approximation is discussed in detail in [13]. Here, we write the important equations that give the correlation function.

First, we have to calculate the coherent component to determine the wave propagating within the snow medium. This can be done using the transmission line model [14]. After we get the "up-going (ψ_u)" and "down-going wave (ψ_d)" in the snow layer, we can calculate the ACF/FCF due to the surface 1 (air-snow interface) and surface 2 (snow-ground interface). The transmission line equation is given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1/Z_1 & -1/Z_1 \end{bmatrix} \begin{bmatrix} \psi_{d1} \\ \psi_{u1} \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \exp(-iq_2h_2) & \exp(iq_2h_2) \\ \exp(-iq_2h_2)/Z_2 & -\exp(iq_2h_2)/Z_2 \end{bmatrix} \begin{bmatrix} \psi_{d2} \\ \psi_{u2} \end{bmatrix}$$

$V_1 = E_o(1+R_s)$, $I_1 = \dfrac{E_o}{Z_1}(1-R_s)$, $V_2 = E_oT_s$, $I_2 = \dfrac{E_oT_s}{Z_3}$,

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$ where $q_m = k_m \cos\theta_m$,

$A_m = D_m = \cos q_m h_m$, $B_m = jZ_m \sin q_m h_m$, $C_m = \dfrac{j\sin q_m h_m}{Z_m}$

$R_s = \dfrac{A + B/Z_3 - Z_1(C + D/Z_3)}{A + B/Z_3 + Z_1(C + D/Z_3)}$, $T_s = \dfrac{2}{A + B/Z_3 + Z_1(C + D/Z_3)}$

For TE waves, $Z_m = \dfrac{\omega\mu_m}{q_m}$ while, for TM waves, $Z_m = \dfrac{q_m}{\omega\mu_m}$.

For first-order Kirchhoff, the scattered wave is

$$E_s = k\cos\theta_s \sqrt{\frac{2\pi}{kR}} \exp(-ikR - i\pi/4) T_{KA1}(\bar{K}, \bar{K}_i) \tag{1}$$

where $T_{KA1}(\bar{K},\bar{K}_i) = H_1 I_{KA1}$, $H_1 = \dfrac{R_1}{4\pi K_z} \bar{N}_1 \cdot (\bar{K} - \bar{K}_i)$,

$I_{KA1} = \int \exp\left(-i(\bar{K}-\bar{K}_i)\cdot\bar{r}_1\right) I_{inc}^{1/2} S(\theta_i,\theta_s)\, dx_1$, I_{inc} is the illumination function which is assumed to take the form $I_{inc} = \exp(-x^2/L_x^2)$ where, L_x is the illumination length in x-direction. $\bar{N}_1 = \dfrac{\bar{K}-\bar{K}_i}{|\bar{K}-\bar{K}_i|}$, $\bar{K}$ and $\bar{K}_i$ are the scattered and incident wave vectors given by

$$\begin{aligned} \bar{K} &= K\hat{x} + K_z\hat{z} = k(\sin\theta_s\hat{x} + \cos\theta_s\hat{z}) \\ \bar{K}_i &= K_i\hat{x} + K_{iz}\hat{z} = k(\sin\theta_i\hat{x} - \cos\theta_i\hat{z}) \end{aligned}. \tag{2}$$

The function R_1 is given by

$$R_1 = \begin{cases} \dfrac{\cos\theta_{no} - \sqrt{\varepsilon_r - \sin^2\theta_{no}}}{\cos\theta_{no} + \sqrt{\varepsilon_r - \sin^2\theta_{no}}} & \text{(TE)} \\[2ex] \dfrac{\varepsilon_r\cos\theta_{no} - \sqrt{\varepsilon_r - \sin^2\theta_{no}}}{\varepsilon_r\cos\theta_{no} + \sqrt{\varepsilon_r - \sin^2\theta_{no}}} & \text{(TM)} \end{cases} \tag{3}$$

where θ_{no} is the angle between $\bar{N}_1$ and $\bar{K}_i$, ε_r is the dielectric constant of snow in this case.

The correlation of two scattered fields at different angles and frequencies ($E_s(\theta_i,\theta_s)$ and $E_s(\theta_i',\theta_s')$) is given by

$$\begin{aligned} \left\langle E_s(\theta_i,\theta_s) E_s(\theta_i',\theta_s') \right\rangle &= 2\pi \frac{\sqrt{kk'}}{R} \cos\theta_s \cos\theta_s' \\ &\quad \left\langle T_{KA1}(\bar{K},\bar{K}_i) T_{KA1}(\bar{K}',\bar{K}_i') \right\rangle \end{aligned} \tag{4}$$

where

$$\left\langle T_{KA1}(\bar{K},\bar{K}_i) T_{KA1}(\bar{K}',\bar{K}_i') \right\rangle = H_1 H_1'^{*} \Phi_1 \Phi_{s1} \left[S(\theta_i,\theta_s) S(\theta_i',\theta_s') \right]^{1/2}$$

$$\Phi_1 = \exp\left(-\frac{\sigma^2}{2}(v_z - v_z')^2\right)\sqrt{\frac{\pi}{v_z v_z'}}\left(\frac{l}{\sigma}\right)\exp\left(-\frac{v_c^2 l^2}{4 v_z v_z' \sigma^2}\right),$$

$$\Phi_{s1} = \left(\sqrt{\pi} L_{eq}\right)\exp\left(-\frac{v_d^2 L_{eq}{}^2}{4\pi}\right),$$

$$\bar{K} - \bar{K}_i = v\hat{x} + v_z\hat{z},\ \bar{K}' - \bar{K}_i' = v'\hat{x} + v_z'\hat{z},\ v_c = \tfrac{1}{2}(v + v'),$$

$$v_d = (v - v').$$

The angular shadowing functions S are

$$S(\theta_k) = \left(1 + erf(v_k)\right)\left(1 - \exp(-F_k)\right)\frac{1}{2F_k}$$

$$S(\theta_1, \theta_2) = \left[1 - \exp\left(-(F_1 - F_2)\right)\right]\frac{erf(v_1) + erf(v_2)}{2(F_1 + F_2)}$$

where $F_k = \frac{1}{2}\left[\frac{\exp(-9v_k^2/8)}{\sqrt{3\pi}v_k} + \frac{\exp(-v_k^2)}{\sqrt{\pi}v_k} - \left(1 - erf(v_k)\right)\right]$,

$v_k = \frac{|\tan\theta_k|}{2\sigma/l}$, and erf denotes the error function.

IV. Volume Scattering

For volume scattering, we apply the Rayleigh scattering approximation because the particles are very small compared to the wavelength. The ACF/FCF are given by

$$\left\langle \psi_v(\theta_{s1}, \omega_1)\psi_v^*(\theta_{s2}, \omega_2)\right\rangle = \frac{f(\hat{o}_1, \hat{i}_1) f^*(\hat{o}_2, \hat{i}_2)}{(4\pi)^2 R_1 R_2}\psi_{i1}\psi_{i2}^* \times \exp(-jk_1R_1 + jk_2R_2) I \qquad (5)$$

where

$$I = \rho V_c \exp\left(-\frac{v_{dx}^2 L_x^2}{4} - \frac{v_{dy}^2 L_y^2}{4}\right)\operatorname{sinc}\left(\frac{v_{zd} h_2}{2}\right)\exp\left[j(\mathbf{q}_1 - \mathbf{q}_2)\bullet\mathbf{r}_c\right],$$

$$\begin{aligned}\mathbf{q_1} &= k_1(\hat{i}_1 - \hat{o}_1) = v_{1x}\hat{x} + v_{1y}\hat{y} + v_{1z}\hat{z}\\ &= k_1\left[\sin\theta_{1o}\cos\phi_{1o} - \sin\theta_{1i}\cos\phi_{1i}\right]\hat{x}\\ &\quad + k_1\left[\sin\theta_{1o}\sin\phi_{1o} - \sin\theta_{1i}\sin\phi_{1i}\right]\hat{y}\\ &\quad + k_1\left[\cos\theta_{1o} + \cos\theta_{1i}\right]\hat{z},\end{aligned}$$

$$\begin{aligned}\mathbf{q_2} &= k_2(\hat{i}_2 - \hat{o}_2) = v_{2x}\hat{x} + v_{2y}\hat{y} + v_{2z}\hat{z}\\ &= k_2\left[\sin\theta_{2o}\cos\phi_{2o} - \sin\theta_{2i}\cos\phi_{2i}\right]\hat{x}\\ &\quad + k_2\left[\sin\theta_{2o}\sin\phi_{2o} - \sin\theta_{2i}\sin\phi_{2i}\right]\hat{y}\\ &\quad + k_2\left[\cos\theta_{2o} + \cos\theta_{2i}\right]\hat{z},\end{aligned}$$

$v_{xd} = v_{1x} - v_{2x}$; $v_{yd} = v_{1y} - v_{2y}$; $v_{zd} = v_{1z} - v_{2z}$, ρ is the particle density, $\mathbf{r}_c$ is the vector coordinate of the center of the volume, $V_c = L_x L_y h_2$ is the common volume. ψ_{i1} and ψ_{i2} are the coherent field incident at the top of snow layer for wave 1 and wave 2, respectively. The scattering amplitude is $f(\hat{o}, \hat{i}) = \frac{k^2}{4\pi}\left|\frac{3(\varepsilon_r - 1)}{(\varepsilon_r + 2)}\right| V_p \sin\chi$ where V_p is the volume of the inclusions, χ is the angle between $\hat{o}$ and $\hat{e}$ (electrical field vector), L_x and L_y are the illumination length in the x-direction and y-direction, respectively. The function I gives the "*Memory dot*" characteristic to the volume scattering correlation function. Note that the correlation functions for volume scattering are calculated based on three-dimensional geometry for the spherical particles. On the other hand, the correlation functions for surface scattering explained in the previous section are based on two-dimensional geometry, which is not entirely matched. In our future work, we will include the correlation function for surface scattering based on three-dimensional geometry.

V. Results

There is an important distinction between the rough surface correlation function and the volume scattering correlation function. The rough surface correlation function has a *memory line* feature, while the volume scattering correlation function exhibits a *memory dots* feature. They are the result of the phase matching conditions. The phase of the rough surface correlation function provides information about the thickness of the layer since it inherits the periodicity of the coherent wave from the transmission line model. On the other hand, the phase of the correlation function from volume scattering depends on the positions of the inclusions, which is random. Therefore, it does not provide useful information about the snow thickness. Fortunately, the effect from volume scattering can be suppressed by choosing appropriate combinations of frequencies and angles for the correlation function. Due to the *memory line* and *memory dots* feature, we can choose a location where the memory line is the dominant term as shown in Fig. 2.

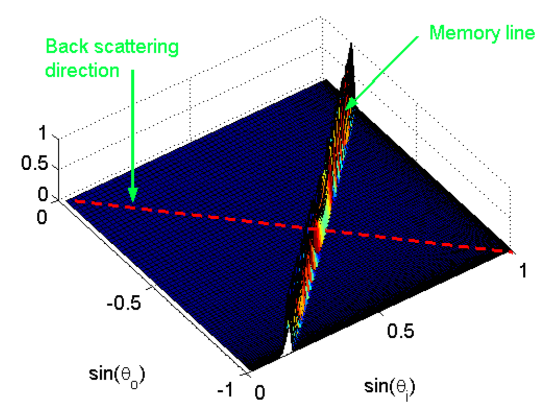

Figure 2. Memory line

Here, the memory line is very dominant, while the memory dots are so small that they do not show up. Therefore, almost all the correlation function comes from the contribution of surface scattering. We are interested in the backscattering correlation so the angle chosen is the intersection of the memory line and the back scattering direction line.

To get the phase matching condition for surface scattering ($v_d = 0$), we use the following frequencies and angles:

Wave 1: 5.3 GHz with incident angle of 30 degrees and

Wave 2: 4.6 GHz with incident angle of 35 degrees.

Both waves are observed at backscattered angles. The phase of ACF/FCF is plotted against the snow depth in Fig. 3. It shows that the phase of ACF/FCF is a linear function of snow depth. Thus, this ACF/FCF method shows potential for snow depth estimation. The results also show that the slope of the phase of ACF/FCF depends on the snow class. This is expected because snow of different classes has different bulk dielectric constants. Therefore, it is necessary to know the density of the snow to be able to use this method for snow depth estimation.

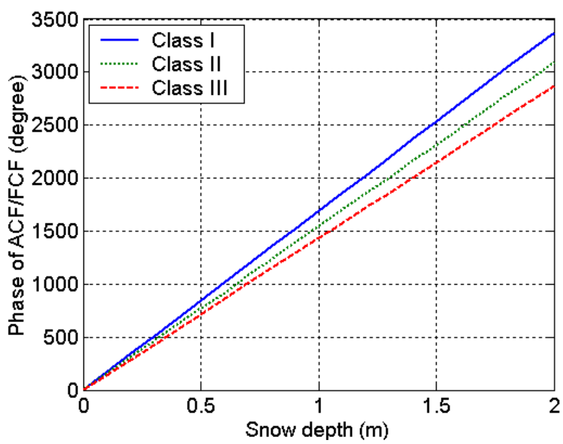

Figure 3. Phase of ACF/FCF as a function of snow depth

VI. CONCLUSIONS

We introduce the correlation function method for potential depth estimation of dry snow-cover on the ground. The model illustrated here is a one-dimensional three-layered medium, which consists of air, snow, and ground. The bulk dielectric constants of snow are calculated based on the QCA-CP mixing formula. We consider three classes of snow, representing different densities and grain sizes. Coherent waves are calculated based on the transmission line model. Correlation functions for surface scattering and volume scattering are discussed. The surface scattering correlation function is based on the first-order Kirchhoff approximation and the volume scattering correlation function is based on multiple scattering theory for discrete dense media using QCA-CP. We exploit the *memory line* and *memory dots* properties of the correlation function to suppress the volume scattering. The phase information from ACF/FCF of surface scattering reveals features that can be used for snow-depth estimation.

ACKNOWLEDGMENT

This work is partially supported by the Jet Propulsion Laboratory.

REFERENCES

[1] J. C. Shi and J. Dozier, "Estimation of snow water equivalence using SIR-C/X-SAR, part I: Inferring snow density and subsurface properties," IEEE Transactions on Geoscience and Remote Sensing, vol. 38, pp. 2465-2474, 2000.

[2] J. C. Shi and J. Dozier, "Estimation of snow water equivalence using SIR-C/X-SAR, part II: Inferring snow depth and particle size," IEEE Transactions on Geoscience and Remote Sensing, vol. 38, pp. 2475-2488, 2000.

[3] H. T. Ewe, H. T. Chuah, and A. K. Fung, "A backscatter model for a dense discrete medium: analysis and numerical results," Remote Sensing of Environment, vol. 65, pp. 195-203, 1998.

[4] J. R. Kendra, K. Sarabandi, and F. T. Ulaby, "Radar measurements of snow: Experiment and analysis," IEEE Transactions on Geoscience and Remote Sensing, vol. 36, pp. 864-879, 1998.

[5] R. M. Narayanan and R. E. Mcintosh, "Millimeter-Wave Backscatter Characteristics of Multilayered Snow Surfaces," IEEE Transactions on Antennas and Propagation, vol. 38, pp. 693-703, 1990.

[6] F. Papa, B. Legresy, N. M. Mognard, E. G. Josberger, and F. Remy, "Estimating terrestrial snow depth with the Topex-Poseidon altimeter and radiometer," IEEE Transactions on Geoscience and Remote Sensing, vol. 40, pp. 2162-2169, 2002.

[7] T. Strozzi, A. Wiesmann, and C. Matzler, "Active microwave signatures of snow covers at 5.3 and 35 GHz," Radio Science, vol. 32, pp. 479-95, 1997.

[8] M. Bernier and J.-P. Fortin, "The potential of times series of C-Band SAR data to monitor dry and shallow snow cover," IEEE Transactions on Geoscience and Remote Sensing, vol. 36, pp. 226-43, 1998.

[9] J. R. Kendra and K. Sarabandi, "A hybrid experimental theoretical scattering model for dense random media," IEEE Transactions on Geoscience and Remote Sensing, vol. 37, pp. 21-35, 1999.

[10] M. T. Hallikainen, F. T. Ulaby, and M. Abdelarazik, "Dielectric properties of snow in the 3 to 37 GHz range," IEEE Transactions on Antenna and Propagation, vol. AP-34, no. 11, November 1986.

[11] F.T. Ulaby, R. K. Moore, and A. K. Fung, *Microwave Remote Sensing: Active and Passive*, vol. 3, Norwood, MA: Artech House, 1986.

[12] L. Tsang and J. A. Kong, *Scattering of Electromagnetic Waves: Advanced Topics*, New York: John Wiley & Sons, 2001, pp. 197-319.

[13] Y. Kuga, C. T. C. Le, A. Ishimaru, and S. L. Ailes, "Analytical, experimental, and numerical studies of angular memory signatures of waves scattered from one-dimensional rough surfaces," IEEE Transactions on Geoscience and Remote Sensing, vol. 34, pp. 1300-7, 1996.

[14] A. Ishimaru, Electromagnetic *Wave Propagation, Radiation, and Scattering*. Englewood Cliffs, NJ: Prentice Hall, 1991.