THE FEATURES OF THE ANGULAR SPECTRUM OF MULTIPLY SCATTERED RADIATION IN TURBULENT COLLISIONAL MAGNETIZED PLASMA

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Introduction

The analysis of statistical characteristics of electromagnetic waves of the small amplitude passing through a plane layer of turbulent plasma in natural or laboratory conditions is actual in many practical applications. Last years it has been found that losses of energy of the electromagnetic field, caused by collisions between particles of plasma, can lead not only decrease of amplitude of waves at removal from boundary of a layer, but also to substantial distortion of the angular distribution of radiation at multiple scattering on smooth random inhomogeneities of medium concentration [1,2]. The influence of collisions of particles in turbulent plasma on the angular distribution of multiply scattered short waves is studied in the given paper on the basis of complex geometrical optics approximation at any angles of both refraction and inclination of a magnetic field.

Statistical characteristics of multiply scattered radiation

Let plane electromagnetic wave with frequency ω is incident from vacuum on semi-infinite layer of collisional magnetized turbulent plasma. We shall choose the Cartesian system of coordinates so that XY plane is boundary of two medium and z-axis is directed inside plasma layer. Plane XZ coincides with the plane formed by a vector of an external magnetic field $\mathbf{B}_0$ and a wave vector of a refracted wave $\mathbf{k}$, θ_0 is an angle between a magnetic field and z-axis. Electron concentration in plasma layer is $N(\mathbf{r}) = N_0 + N_1(\mathbf{r})$, where N_0 is constant component, and $N_1(\mathbf{r})$ is random function of spatial coordinates describing fluctuations of electronic concentration ($N_1 << N_0$). Let's assume that the characteristic size of inhomogeneities substantially exceeds the wavelength λ that allows using geometrical optics approximation. For propagating normal electromagnetic wave in plasma with smooth inhomogeneities it is possible to write the eikonal equation [3,4] $\tilde{\mathbf{k}}^2 = (\omega^2 / c^2)\tilde{n}^2$, where $\tilde{\mathbf{k}}(\mathbf{r}) = -\nabla\tilde{\varphi}$ is a complex wavevector in the given point of space, $\tilde{\varphi}$ is a complex phase of a wave, and $\tilde{n}^2 = \tilde{n}^2(N(\mathbf{r}), \omega, \tilde{\mathbf{k}})$ is a complex refractive index. Incident on a layer from vacuum the plane wave after refraction on boundary a wave becomes extraordinary. Statistical characteristics of scattered field basically are determined by fluctuations of a complex phase of extraordinary plane wave in the case of small–angle scattering [1,2]. Let's expand the phase and the wave vector into a series: $\tilde{\mathbf{k}} = \tilde{\mathbf{k}}_0 + \tilde{\mathbf{k}}_1(\mathbf{r}) + \ldots$, $\tilde{\varphi} = \tilde{\varphi}_0 + \tilde{\varphi}_1 + \ldots$ It is possible to obtain generalized on a case of absorbing medium the expression (hereinafter an index "0" at a component of a wave vector $\tilde{\mathbf{k}}_0$ it is omitted for brevity)

$$\frac{\partial\tilde{\varphi}_1}{\partial z} + \frac{\partial\omega/\partial k_x}{\partial\omega/\partial\tilde{k}_z}\frac{\partial\tilde{\varphi}_1}{\partial x} = -\omega\left(\frac{\partial(\tilde{n}\omega)}{\partial\omega}\frac{\partial\omega}{\partial\tilde{k}_z}\right)^{-1}\frac{\partial\tilde{n}}{\partial N_0}N_1 \tag{1}$$

and the expression for transversal correlation function of the phase

$$R_{\tilde{\varphi}}(\rho_x,\rho_y,z) = 2\pi\tilde{\alpha}^2\int_{-\infty}^{\infty}d\kappa_x\int_{-\infty}^{\infty}d\kappa_y\frac{\exp(i\kappa_x\rho_x + i\kappa_y\rho_y + 2\kappa_x\gamma z)}{2\kappa_x\gamma}\times$$

$$\times\left[1-\exp(2\kappa_x\gamma z)\right]\Phi(\kappa_x,\kappa_y,-\beta\kappa_x)\ , \tag{2}$$

where $\tilde{\alpha} = -\omega\left(\frac{\partial(\tilde{n}\omega)}{\partial\omega}\frac{\partial\omega}{\partial\tilde{k}_z}\right)^{-1}\frac{\partial\tilde{n}}{\partial N_0}$, $\Phi(\kappa_x,\kappa_y,\kappa_z)$ is the three–dimensional spatial power spectrum of statistically homogeneous electron concentration fluctuation, and $\partial\tilde{k}_z/\partial k_x = \beta + i\gamma$. The imaginary part, equal γ plays an essential role as in (2) enters into an exponent and exerts accumulate with depth *z* influence on statistical characteristics of phase fluctuations. In case of multiple scattering correlation function of complex field and spatial (angular) spectrum of scattered field, which is Fourier transform of correlation function of a field, has the Gaussian form and are presented in [6,7]. In particular

$$\Delta k_x = \frac{2\pi}{\gamma}\frac{\alpha^2}{k_{z2}^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\,\Phi_p(k_x,k_y,-\beta k_x)\left[\exp(2k_x\gamma z)-1\right],$$

$$<k_x^2> = \frac{2\pi}{\gamma}\frac{\alpha^2}{k_{z2}^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\,\Phi_p(k_x,k_y,-\beta k_x)k_x\left[\exp(2k_x\gamma z)-1\right], \tag{3}$$

where $\Delta\kappa_x$ determines the displacement of its maximum causing by random inhomogeneities, $<\kappa_x^2>$ is determined the width of this spectrum in XZ plane. The spectrum Φ should be properly normalized on value of the root–mean–square fluctuations of electron concentration $4\pi\int_0^{\infty}\kappa^2\Phi(\kappa_x,\kappa_y,\kappa_z=0)d\kappa = <N_1^2>$. However $<N_1^2>$, as a rule, is obtained from natural experiments by an indirect route and can depend in rather difficultly on a plenty of external factors. From formula (3) follows that anomalous broadening and displacement of the centre of gravity take place at $\gamma \neq 0$, and, the contribution both inclined incidence of radiation, and anisotropy of medium take part in this imaginary part.

Components of a wavevector $\tilde{\mathbf{k}} = (\omega/c)\{p,\ 0,\ \tilde{q}\}$ at inclined incident wave on the boundary of magnetized plasma satisfy the equation [3]:

$$a_4\tilde{q}^4 + a_3\tilde{q}^3 + a_2\tilde{q}^2 + a_1\tilde{q} + a_0 = 0\ , \tag{4}$$

where coefficients are expressed through the parameter p as:

$$a_0 = \left[\left(1-p^2\right)\left(1-is\right)-v\right]\left\{\left[\left(1-p^2\right)\left(1-is\right)-v\right]\left(1-v-is\right)-\left(1-p^2\right)u\right\}-$$

$$-\left(1-p^2\right)p^2u_x v, \quad a_1 = -2\left(1-p^2\right)pv\sqrt{u_x u_z},$$

$$a_2 = -2\left(1-is\right)\left\{\left[\left(1-p^2\right)\left(1-is\right)-v\right]\left(1-v-is\right)-\left(1-p^2\right)u\right\}+$$

$$+v\left[p^2u_x-\left(1-p^2\right)u_z-u\right], \quad a_3 = 2pv\sqrt{u_x u_z},$$

$$a_4 = (1-is)[(1-is)^2-u]-v[(1-is)^2-u_z],$$

$u=\omega_B^2/\omega^2$, $v=\omega_p^2/\omega^2$ and $s=v_{eff}/\omega$ are non-dimensional parameters, $u_x = u\sin^2\theta_0$, $u_z = u\cos^2\theta_0$, ω_p is the cyclic frequency of own plasma oscillations, ω_B is the electron's gyrofrequency, v_{eff} is the effective collisional frequency between electrons and other plasma particles. The analytical solution of this equation is rather complicated and therefore its further analysis has been carried out numerically. As a result it is possible to construct the curves of dependence of an imaginary part $\tilde{q}$ $\left(\mathrm{Im}(\tilde{q})\right)$ versus parameter p for both normal waves. Then $\gamma = \mathrm{Im}\,\partial\tilde{q}/\partial p$ will be determined as a tangent of an angle of an inclination of a tangent to a curve with respect to an abscissa axis. The problem is to determine the angle of refraction and the angle of inclination of magnetic field for concrete values of various parameters of plasma, at which for the given type of a wave γ will be zero and, on the contrary, where this value substantially differs from zero. Tends of parameter γ to zero means, that influence of absorption on the angular spectrum in geometrical optics approximation is absent i.e. the influence of inclined refraction and anisotropy compensate each other. The effect of compensation has been marked in [6,7]. At absence of compensation with increasing of γ anomalous effects of influence of absorption on the angular spectrum will be more brightly. For confirmation of this suggestion the numerical calculation of the task has been carried out using Monte–Carlo method. The "Classical" variant of Monte–Carlo method takes into account absorption through so-called "probability of a survival" Λ, which connects scattered and absorption characteristics of an isotropic medium. For isotropic medium $\Lambda = \sigma_s/(\sigma_s+\sigma_a)$, where σ_s and σ_a are extinction coefficient of scattering and absorption respectively. Setting this probability, wavelength and refraction coefficient for isotropic media it is possible to obtain the exhaustive data for modelling their scattering properties. The extinction coefficient of absorption in collisional magnetized plasma can be found using simple formula $\sigma_a = 2\omega c^{-1}\,\mathrm{Im}\;\tilde{n}(\theta)$. For our case the imaginary part of refraction coefficient is presented in [6,7]. Numerical calculations were carried out for $\omega_H \approx 8.8\cdot 10^6$ rad/sec, $\omega_p \approx 10^7$ rad/sec and $v \approx 10^5$ sec^{-1}, that corresponds to the average parameters of F-layer of the ionosphere. The Figure evidently shows, that for each normal wave there is an own direction of compensation, where $\mathrm{Im}\left(\partial\tilde{q}/\partial p\right)=0$. The effect of compensation can be found out in a quiet ionosphere. Besides in case of strong artificial or natural perturbation of an ionosphere when plasma concentration and number of scattering between particles sharply grows this effect can become quite observable.

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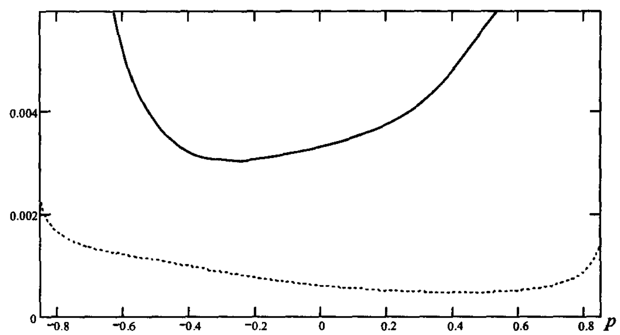

The dependences $\mathrm{Im}(\tilde{q}(p))$ for extraordinary (solid line) and ordinary (dotted line) normal waves in plasma with parameters $u=0.22$; $v=0.28$ and $s=0.0053$ at an angle of inclination of a magnetic field $\theta_0=40$ degree. The angle between the direction of compensation and a normal to the layer makes ≈ 23.5 degree ($p\approx 0.4$) for an ordinary wave and ≈ -14.5 degree for extraordinary wave ($p\approx -0.25$).