# IMAGING THROUGH DIFFUSIVE MEDIUM BY PHOTON DENSITY WAVES

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## ABSTRACT

Photon density wave is the modulated optical intensity wave which propagates through the diffuse medium and exhibits the amplitude and phase variations. It satisfies the time-dependent radiative transfer equation and is similar to temperature wave field, and has been used in medical imaging. This paper discusses the basic characteristics of the photon density wave and gives exact numerical solution for plane-parallel medium, the degree of polarization, cross-polarization discrimination, and the point spread function to show the superior imaging properties over conventional imaging techniques.

## INTRODUCTION

Photon density waves have been investigated and used for optical imaging in biological medium. Photon density waves make use of the modulated intensities with the modulating frequency, typically, of the order of 100 MHz in medical optics. The modulated intensity propagates through the scattering medium and exhibits the amplitude and phase variations, much as the conventional wave field. However, unlike the wave field which satisfies the wave equation, this photon density wave satisfies the frequency-dependent radiative transfer equation. It is similar to the temperature field which can also behave as a wave, and the electromagnetic field in highly conducting medium [1]-[9].

In medical optics, the photon density waves have recently been investigated based on the diffusion approximation of the radiative transfer equation. The diffusion equation involves the scattering and absorption coefficients, the transport and the mean free path length, and the diffusion coefficient. The density wave exhibits the wave characteristics much as the ordinary wave, but its wavelength is proportional to the square root of the product of the modulation wavelength and the mean transport path length. However, its attenuation path length is close to the wavelength of the density wave. This is similar to the electomagnetic field propagation in conducting medium where the skin depth is close to the wavelength in the medium.

Even though the photon density wave has been extensively studied using the diffusion approximation, the exact solution should be based on the time-dependent radiative transfer theory. This paper presents such an analysis and shows that the photon density wave gives superior resolution capabilities than other conventional imaging techniques through scattering medium. It should be noted that the diffusion approximation is useful for many problems of complex geometries, while the radiative transfer equation has been solved for a limited number of simple geometries. One such problem is the transmission and reflective of plane wave incident upon a plane parallel scattering medium. In this paper, we consider the photon density wave propagating through a plane parallel diffuse medium based on the complete time-dependent vector radiation transfer theory. We consider the pulsed incident wave with circular and linear polarizations and the imaging properties of the focusing system.

## FORMULATION OF THE RADIATION TRANSFER THEORY FOR THE PULSED PHOTON DENSITY WAVE

We first consider a narrow band radiative transfer equation. The incident intensity is a modulated pulse and thus the temporal Fourier transform of the radiative transfer equation is considered. It is soon found that the numerical solution is unstable because of the high spatial frequency components. The equation is then modified to yield stable numerical solutions. This equation is then solved by using the discrete ordinates method, and the final results are Fourier-transformed back to the time domain. The Mueller matrix is calculated based on Mie scattering theory for spherical dielectric particles [10]-[12].

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## LINEAR AND CIRCULAR POLARIZATIONS

The photon density wave is conventionally represented by scalar intensities. However, in our vector radiative transfer, we make use of the 4X1 Stokes vector and 4X4 Mueller matrix, and thus the complete polarization characteristics can be obtained [13] as shown in Fig. 1.

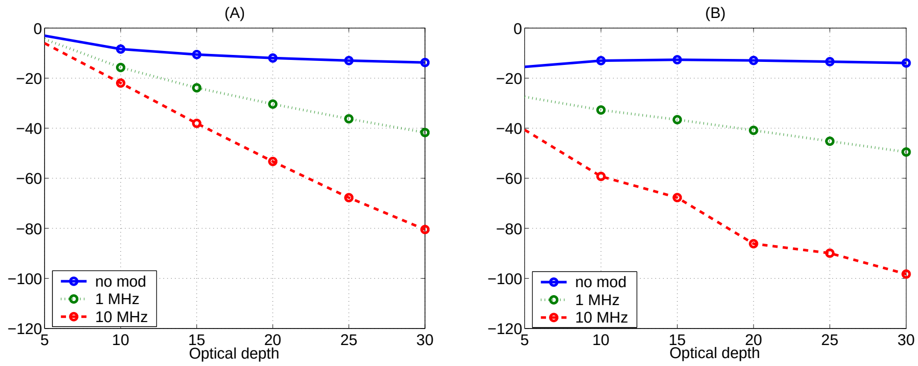


Fig 1. Diffuse intensity of CW photon density waves with different modulation frequencies as a function of optical depth (A) circular co-polarization and (B) circular-cross-polarization

## POINT SPREAD FUNCTION (PSF) AND MODULATION TRANSFER FUNCTION (MTF)

As the photon density wave propagates through the random medium, it experiences multiple scattering and the angular spectrum spreads out. The transmitted Stokes vector is then Fourier - Transformed to obtain the mutual coherence function which is applied to the imaging system. The imaging system consisting of a lens (or reflector) focuses the transmitted intensity and forms an image. The imaging properties can be expressed by "Point Spread Function" (PSF) or its Fourier Transform the "Modulation Transfer Function" (MTF). We can also use the Degree of Polarization (DOP), Cross-polarization Discrimination (XPD), and the “contrast” to quantify the qualities of the image. It is shown that the diffuse component of the photon density wave is smaller than the diffuse component of the intensities in the conventional imaging system, and thus the photon density wave gives a higher resolution than the other conventional imaging techniques [14]-[16]. Fig.2 shows the polarization characteristics of the photon density wave.

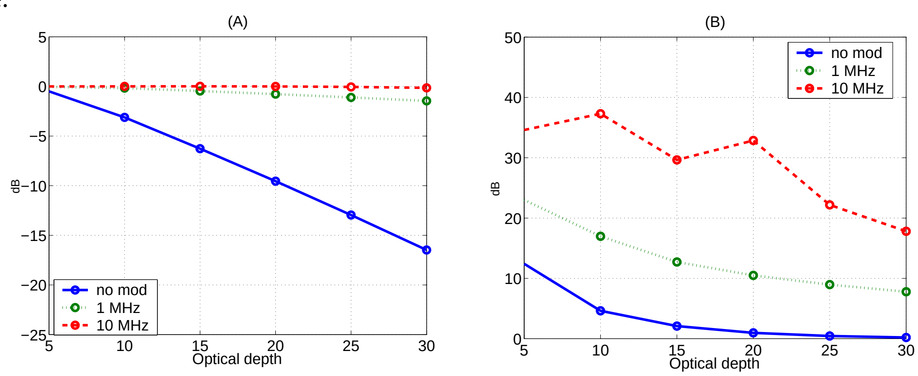


Fig 2. Polarization characteristics of photon density waves with different modulation frequencies as functions of optical depth (A) DOP of circular polarization and (B) XPD of circular polarization

## NUMERICAL EXAMPLES AND COMPARISONS WITH OTHER IMAGING TECHNIQUES

We have conducted numerical studies of PSF, DOP, XPD, and contrast for the particle sizes close to a wavelength and much greater than a wavelength. The image system in consideration is shown in Fig. 3. We also calculated the imaging properties using the conventional Stokes vector formulations, and compared the results of the photon density imaging with other techniques such as the Cross-Polarization Intensity Subtraction method (CPIS), and the Off-Axis Intensity Subtraction method (OAIS). It is shown in Fig. 4 that the photon density wave imaging gives higher resolution than other techniques.

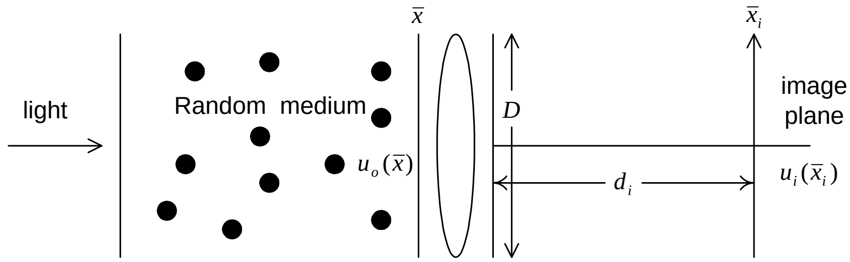


Fig 3. An imaging system

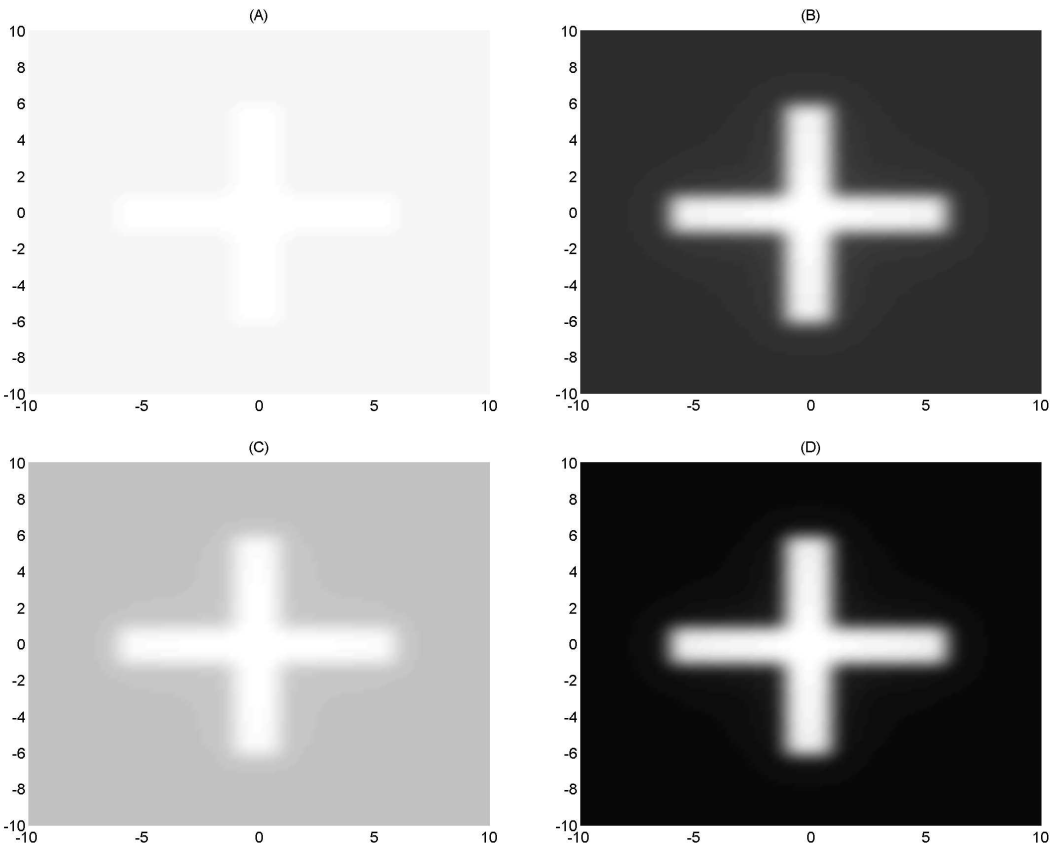


Fig. 4. The linear polarized cross images through random media of optical depth 30 using (A) co-polarized wave, (B) CPIS, (C) OAIS, and (D) 1 MHz photon density. The angular sizes are in microradians with the aperture size of 1 m.

**CONCLUSIONS**

We presented the use of photon density waves for imaging in scattering mediums. We solved the time-dependent vector radiative transfer, rather than its scalar approximation of diffusion equation. The transmitted Stokes vector is then Fourier-transformed to obtain the mutual Coherence Function, which is then applied to the focusing imaging system to obtain PSF(point spread function) and MTF(Modulation transfer function). The results are compared with other imaging techniques and show that the photon density waves give higher image resolution than other imaging techniques.

## REFERENCES


[1] B.J. Tromberg , L.O. Svaasand , T.-T. Tsay, and R.C. Haskell, “Properties of photon density waves in multiple-scattering media,” *Applied Optics*, Vol. 32, pp. 607-616, 1993.

[2] S. Fantini and E. Gratton, “Fluorescence photon-density waves in optically diffusive media,” *Optical Communication*, Vol. 173, pp. 73-79, 2000.

[3] J.B. Fishkin, S. Fantini, M.J. VandeVen, and E. Gratton, “Gigahertz photon density waves in a turbid medium: Theory and experiments” *Physical Review E*, Vol. 53, pp. 2307-2319, 1996.

[4] C.L. Matson and H. Liu, “Back propagation in turbid media,” *Journal of Optical Society of America A*, Vol. 16, pp. 1254-1265, 1999.

[5] C.L. Matson and H. Liu, “Analysis of the forward problem with diffuse photon density waves in turbid media by use of a diffraction tomography model,” *Journal of Optical Society of America A*, Vol. 16, pp. 455-466, 1999.

[6] C.L. Matson and H. Liu, “Resolved object imaging and localization with the use of a backpropagation algorithm,” *Optical Express*, Vol. 6, pp. 168-174, 2000.

[7] C. L. Matson, “A diffraction tomographic model of the forward problem using diffuse photon density waves,” *Optical Express*, Vol. 1, pp. 6-11, 1997.

[8] C.L. Matson, N. Clark, L. McMackin, and J.S. Fender, “Three-dimensional tumor localization in thick tissue with the use of diffuse photon-density waves,” *Applied Optics*, Vol. 36, pp. 214-220, 1997.

[9] C.L. Matson, N. Clark, L. McMackin, and J.S. Fender, “Three-dimensional tumor localization in thick tissue using measurements in a plane,” *OSA Trends in Optics and Photonics on Advances in Optical Imaging and Photon Migration*,Vol. 2, pp. 179-181, 1996.

[10] A. Ishimaru, S. Jaruwatanadilok, and Y. Kuga, “Polarized Pulse Waves in Random Discrete Scatterers,” *Applied Optics*, Vol. 40, pp. 5495-5502, 2001.

[11] A.D. Kim, S. Jaruwatanadilok, A. Ishimaru, and Y. Kuga, “Polarized light propagation and scattering in random media,” *SPIE*, Vol. 4257, pp.90-100, 2001.

[12] A. Ishimaru, *Wave Propagation and Scattering in Random Media*, IEEE Press, New York, 1997.

[13] R.L.-T. Cheung and A. Ishimaru, “A Transmission backscattering and depolarization of waves in randomly distributed spherical particles,” *Applied Optics*, Vol. 21,pp. 3792-3798, 1982.

[14] S. Jaruwatanadilok, A. Ishimaru, and Y. Kuga Y, “Cross-polarization intensity subtraction technique for imaging through random scattering media,” *unpublished*.

[15] M.C. Roggemann and B. Welsh, *Imaging Through Turbulence*, CRC Press, New York, 1996.

[16] H.C. Van de Hulst, *Light Scattering by Small Particles* , Dover, New York, 1997.