Green's Function for Rough Surface with Dirichlet, Neumann, and Impedance Boundary Conditions

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Abstract

This paper presents an analytical theory of rough surface Green's functions based on the extension of the diagram method of Bass, Fuks, and Ito with the smoothing approximation used by Watson and Keller. The method is a modification of the perturbation method and is applicable to rough surfaces with small rms height. But the range of validity is considerably greater than the conventional perturbation solutions. We consider one-dimensional rough surfaces with Dirichlet, Neumann, and impedance boundary conditions. The coherent Green's function is obtained from the smoothed Dyson's equation by using a spatial Fourier transform. The mutual coherence function for the Green's function is obtained by the first-order iteration of the smoothing approximation applied to the Bethe-Salpeter equation in terms of a quadruple Fourier transform. These integrals are evaluated by the saddle-point technique. The equivalent bi-static cross section per unit length of the surface is compared to the conventional perturbation method and Watson-Keller's result. With respect to Watson-Keller's result, it should be noted that our result is reciprocal while the Watson-Keller result is non-reciprocal. Included in this paper is a discussion of the specific intensity at a given

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observation point. The theory developed will be useful for the RCS signature related problems and LGA (low grazing angle) scattering when both the transmitter and object are close to the surface.

There have been extensive studies made on the rough surface scattering problem. Most studies deal with plane wave incidence and the scattering characteristics are expressed in terms of the cross sections per unit area of the rough surface [1] – [3]. While this is appropriate for moderate angles of incidence (less then 75^{o}), the assumption of plane wave incidence is no longer appropriate when the transmitter and target are near the ocean surface or for LGA scattering. For larger angles of incidence, and scattering near the surface, careful examination of the plane wave assumption is required. For LGA scattering, it has already been pointed out by Barrick [4], [5] that "propagation and scatter become inextricably connected" and "the free-space plane wave description may not suffice". The wave incident at a point on a rough surface is not the direct plane or spherical wave from the transmitter. The incident wave is modified by the rough surface itself. The incident wave at a point on the rough surface is a sum of the free-space plane wave from the transmitter and the scattered wave from the surface. In this paper, we consider the radiation from a point source located at any point near the rough surface, and thus the field on the surface is the total field.

In recent years, several numerical Monte-Carlo techniques have been developed to obtain numerical solutions to the rough surface scattering problem [5]. While this is an excellent approach to the study of rough surface scattering, when the grazing angle becomes small, extremely large surface areas are required to properly take into account the large footprint area. Thus fast high performance computers are required for solutions. The rough surface Green's function is analytical, and the computer requirement is reduced. Fast analysis of the rough surface effects is possible. Therefore, it is important to consider problems in which the rough surface correction of scattering from near-surface objects must be included.

We present an analytical theory of rough surface Green's function for the one-dimensional rough surface. This provides a mathematically simple formulation including the effects of rough surfaces, but it does not include cross-polarization effects. We begin with Green's theorem, and using an equivalent boundary condition, we obtain Dyson's equation for the coherent field which is obtained by using a spatial Fourier transform. If the surface is Dirichlet, the equivalent impedance is zero for the flat surface. However, the impedance is not zero due to the presence of roughness. Also, corresponding to this impedance, there are surface wave poles which give rise to surface wave propagation along the surface. The coherent field is shown to be equivalent to Watson-Keller's results. Next we examine the Bethe-Salpeter equation and obtain the first-order iteration solution once again making use of the spatial Fourier transform. The cross section per unit length is calculated and is shown to be similar to Watson-Keller, but more importantly it is reciprocal. Discussions are also included on power conservation and the specific intensity. This paper discusses the first-order modified perturbation theory of the rough surface Green's function and the far-field approximations. We will discuss the surface wave contributions applicable to the low grazing angle case and the second-order modified perturbation techniques which extend the range of validity of this theory.

TM waves over the impedance rough surface correspond to the classic Sommerfeld problem for rough surface, and the modification of the Sommerfeld pole and Zenneck wave due to surface roughness can be obtained. It is noted that the Sommerfeld attenuation function shows increased attenuation due to surface roughness. The effective surface impedance, Sommerfeld pole location, and Zenneck wave propagation constant have been obtained and numerical examples are given for land and sea rough surfaces. We have also obtained the scattering cross-sections per unit area of the rough surface for Dirichlet, Neumann, and impedance surfaces showing the effects of roughness and surface conductivities. Examples include HH and VV cross-sections of finitely conducting rough surfaces.

This paper presents an analytical theory of the coherent and the incoherent rough surface Green's function for one-dimensional smooth rough surface. The theory is applicable to surfaces with small rms height $k\sigma \leq 1.0$, but the range of validity is much greater than that of the conventional perturbation method. The coherent Green's function was determined from Dyson's equation, and its spatial Fourier transform representation is given. A saddle-point technique was used to evaluate this expression and is given. The mutual incoherent function was calculated based on the Bethe-Salpeter equation, and the general solution based on a spatial Fourier transform is given. This is also evaluated using a far-field asymptotic approximation. The mutual coherence function was then used to calculate the specific intensity. Therefore, the theory should be useful for RCS signature related problems and for LGA scattering when both the transmitter and observation point are close to the surface.

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