Load Pickup and Feeder Reconfiguration of Distribution Networks

Linyue Qiao

A thesis
Submitted in partial fulfillment of the
Requirements for the degree of

Master of Science in Electrical & Computer Engineering

University of Washington

2019

Committee:

Daniel Kirschen

Baosen Zhang

Program Authorized to Offer Degree:
Electrical & Computer Engineering Department

University of Washington

Abstract

Load Pickup and Feeder Reconfiguration of Distribution Networks

Linyue Qiao

Chair of the Supervisory Committee:
Donald W. and Ruth Mary Close Professor: Daniel Kirschen
Keith Nancy Rattie Endowed Career Development Professor: Baosen Zhang
Electrical Computer Engineering Department

The power outage of the distribution networks resulting from natural disasters could bring catastrophic damage to the system as well as millions of financial loss to the people. Restoring distribution networks after these events can help reduce the consequences and therefore have been drawing much attention since 1980s. This thesis focuses on the load pickup problem and the feeder reconfiguration problem in the restoration process. For a single feeder distribution network, system operators try to pick up as many loads as possible after a distribution network power outage. However, the load pickup problem is a combinatorial optimization problem, the computing time of which could be very long for large scale distribution networks. We model it with an ILP formulation and propose an approximation algorithm by casting it into a variant of knapsack problem. Both algorithms are tested and compared intensively with many cases on standard IEEE test feeders. For the multiple feeders interconnected with normally open switches, system operators can alter the system topology by opening and closing switches and shift the loads to pick up them. In this thesis, we formulate the feeder reconfiguration problem as an ILP problem and then perform tests to find out different corner cases that would help us to examine the performance of the ILP solution approach.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

1.1 Harm of Power Outages in Distribution System

Distribution networks are of significant importance to the power system nowadays since they are the very pivot connecting the transmission system and the consumers. One of the popular research subjects about distribution networks is the restoration of the distribution networks after power outages. Most of the power system outages result from natural disasters [55]. They are rare but could result in severe damage to the system and millions of financial loss to nations. The cost for restoration of the damage caused by Hurricane Katrina in 2005 is estimated to be in the range of $ 750 million to $ 1.1 billion [32]. In 2012, the distribution networks were extremely damaged by Hurricane Sandy and approximately 8.35[1] million electric customers suffered from power outages [36]. Hurricane Maria, which hit United States on September 20th 2017, caused 75% of the electricity system still out of service even one month after its landfall and it took about one year for the area to completely recover from the disaster [38]. According to [39], between 2003 and 2012, about 679 widespread power outages occurred because of severe weather and the average annual cost of restoration varied from $ 18 billion to $ 33 billion [51]. Therefore an efficient restoration strategy to supply power to those un-faulted part of the system during the outage period is essential [55].

Distribution system restoration has been extensively studied in many aspects [9], involving expert systems [29], [8], fuzzy logic [27], [24], load restoration [44], network reconfiguration [5] and multiagent systems [37], [48]. This thesis focuses on the load pickup problem and the feeder reconfiguration problem in the restoration process.

---

[1]This consists of 4.78 million customers in PJM, 2.22 million in NYISO, and 1.37 million in ISO-NE.

1.2 Load Pickup

The load pickup problem is also referred as load curtailment problem. After the fault is isolated from the distribution network, the system operators would try to find out how many loads could be picked up by the system or how many loads need to be curtailed from the system while the repairing crew are trying to fix the damaged part. The system operators would like to keep as many loads connected to the network as possible. However, there are many operational constraints that need to be kept in mind. It is well known to us that distribution networks tend to be overdesigned in order to supply more loads in the future [54]. So in most cases, the line capacity will not be the binding constraints. Based on this assumption, the constraints do matters are the capacity of the feeder and the voltage of nodes. Another aspect which also need to be emphasized is that different load has different priority. For example, the hospital is of higher priority than a citizen's house. So the system operators cannot simply use the number of picked up load as their objective.

The load pickup problem has been studied since 1985. Mario V. F. Pereira [41] proposed ranking algorithm based on sensitivity analysis of loss-of-load. Since then, the load pickup problem has been extensively studied in many aspects. A novel modified Viterbi algorithm is proposed in [55], by which the load pickup problem is converted into a shortest path problem. A competitive randomized online algorithm is described in [19]. An algorithm is carried out in [22] which is able to transform the load pickup problem into a knapsack problem. And the author also proposes an approximation algorithm to attain a feasible solution with less time. The algorithms have been tested using IEEE 38-node system and IEEE 123-node system. This gives us a hint that knapsack algorithm would be an efficient method to solve the load pickup problem. After converting load pickup problem into knapsack problem, it could be solved as an optimization problem using integer linear programming (ILP). The objective function of the optimization problem is able to contain both the weight/priority of the loads and the value of the loads. And it is flexible to add constraints or different objectives. However, the shortcoming of the ILP is also obvious. The computing time would be a concern we need to pay attention. As we know, integer programming will not be as fast as linear-programming (LP). Therefore, approximation algorithms which could give out feasible solutions faster than ILP attract many researchers' interest. In this thesis, we use knapsack algorithm to solve the load pickup problem and propose a new approximation algorithm. The algorithm is applied to 38-node system [22], IEEE 123-node system [42] and IEEE 8500-node system [43] and the performance of the approximation algorithm are assessed in chapter 2.

1.3 Feeder Reconfiguration

The definition of feeder reconfiguration was proposed by S. Civanlar [10] in 1988. She referred it as changing the topological structure of distribution feeders by opening or closing the sectionalizing and tie switches. The goal of reconfiguration of distribution networks is to figure out the best structure of the system regarding to power loss and operational performance [12]. So far, the reconfiguration problem has been comprehensively researched in many aspects with various methods. Among them, heuristic methods and direct mathematical methods are two methods mainly used when solving the reconfiguration problems. In most of the mathematical methods, the reconfiguration problem is formulated as a concrete optimization problem and we can obtain the solution directly after we run the program. Previously people would like to use mixed-integer programming (MIP) algorithms to solve the problem since these methods have the following advantages:

- The problems can be solved by MIP solvers
- The robustness and reliability are guaranteed
- It is flexible to add constraints or different objectives

However, the shortcoming of MIP is also obvious: high computational burden. This drawback limits the application of MIP. The heuristic methods stand out for the reduced computational complexity [2]. The heuristic methods tend to solve the problem in several steps [28], [58], [50], [34], [16], [47]. The heuristic algorithms could show us explicit structure of the solving process, however, they bring us the trouble of finding suboptimal solutions. Recently, more and more people use MILP to solve the reconfiguration problem. MILP algorithm covers all MIP's advantages and could get the solution faster than MIP. Rabih A. Jabr [17] and A. Ajaja [4] use MILP to solve the reconfiguration problem in order to reduce the network loss. B. Moradzadeh [35] uses MILP reconfiguration algorithm to solve battery storage problem of distribution system. Ali Khodabakhsh [21] proposes a submodular approach to find out the optimal spanning structure to reduce the system loss. After traversing many correlated papers, we find out that most of the researches about feeder reconfiguration are carried out using heuristic methods. Although some people used mathematical method, most of them aiming at curtailing the system loss. Very few people use mathematical method to solve feeder reconfiguration problem which targets on system restoration. Therefore we use Integer Linear Programming (ILP) to solve the reconfiguration problem of a distribution network including three different feeders. And the objective is to perform the load shifting from one damaged feeder to other feeders.

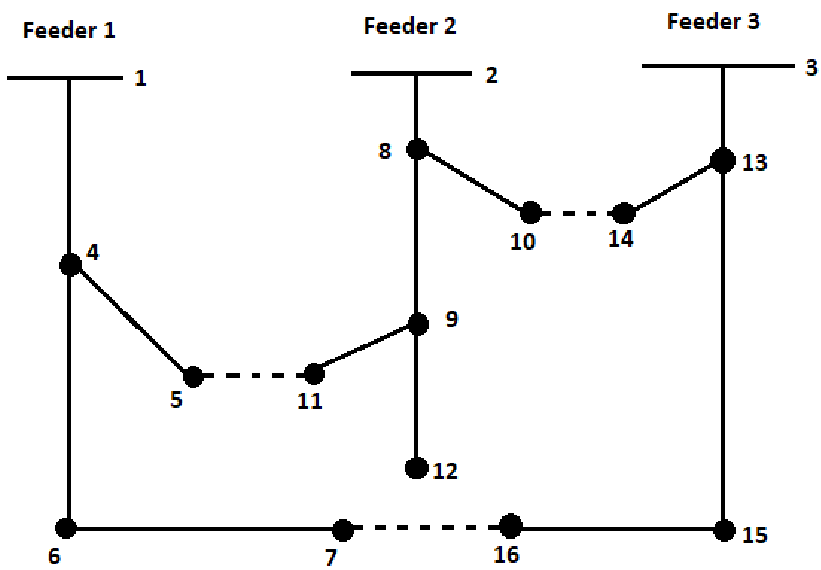

Figure 1.1: 3-feeder 16-node distribution network

Distribution networks are built in meshed structure but normally operated in radial structure for effective coordination of the protection in emergency situations [17]. The distribution network shown in Figure 1.1 is a classical model for distribution networks feeder reconfiguration, which contains three feeders and 16 nodes. Each bold line has a sectionalizing switch which is commonly closed and each dashed line has a tie switch which is commonly open. Let's say there is a fault at Feeder 1 causing the capacity of Feeder 1 decreasing to 60% and Feeder 1 cannot hold all the load previously connected to it. So it is necessary to shift some of the loads from feeder 1 to another two feeders. Meanwhile, we need to make the whole system still obey the voltage constraints, feeder capacity constraints, radiality constraints and line capacity constraints after altering its topological structure. Furthermore, performing the reconfiguration with a minimum number of switch operation is essential to prolong the life of system components. However, there are three main difficulties.

- How to define our objective? We need to think of a method to quantify the number of switch operation.

- How to implement the power flow equations? Since the structure of the distribution network changes, it is impossible to simply use DistFlow equations to characterize the relationship between power flow and the node voltage.

- How to keep the radial structure of each feeder after load shifting? It is necessary to think of a way to implement the radiality constraints.

The detail formulation of this problem is shown in chapter 3. We come up with an ILP algorithm, which could solve this problem efficiently. The algorithm is applied to the three-feeder 16-node system first to examine its accuracy. And a three-feeder 375-node system is also used in test, which is modified from IEEE 123-node system, to examine the algorithm's reliability. The algorithm realizes the function of external reconfiguration between different feeders and internal reconfiguration inside a single feeder.

1.4 Reader's Guide

The rest of this thesis is organized as follows.

- In Chapter 2, the formulation of the load pickup problem and its approximation algorithm are presented. The case studies of 38-node, IEEE 123-node and 8500-node system are also shown in this chapter.

- The algorithm to solve feeder reconfiguration problem is presented in detail in Chapter 3, which is based on some of the work from load pickup problem. Both functions of reconfiguration between different feeders and the internal reconfiguration of one single feeder are realized. The test cases of 3-feeder 16-node system and 3-feeder 375-nodes system are illustrated in detail.

- Chapter 4 concludes the thesis.

- Chapter 5 lays out some possible future work.

Chapter 2

LOAD PICKUP

2.1 Problem Formulation

As mentioned in previous chapter, we would like to pick up as much load as possible without breaking the operation constraints of distribution networks. But different loads have different weight/priority, which means it is not realistic to use the numbers of loads connected to the system as our objective functions. Therefore it is necessary to think of a way to formulate the problem as an optimization problem. What is worth mentioning here is that distribution networks are operated in three phases mode in practice, which makes the formulation of the problem very complicated. So we convert the three-phase systems into single-phase systems and we only consider about single-phase system in this thesis.

There are three main points need be solved in the formulation process.

- How to quantify the objective? We need to find out a way to denote the value of each load so that we could correlate it with the decision variable and get our objective function in a usual optimization problem form.

- How to combine the system constraints with the load pickup problem? As stated previously, the line capacity is not a concern in the load pickup problem. So we only need to pay attention to the voltage constraints of the distribution system. Therefore it is essential to find out a mathematical expression of the node voltage.

- How to formulate unique structure of the distribution networks? Figure 2.1 shows two common structures of distribution networks. In some of the distribution networks (Structure A), the curtailment of one load does not affect the consumers downstream. But in most cases

(Structure B), the curtailment of upstream consumers means all the consumers downstream cannot get the power they need.

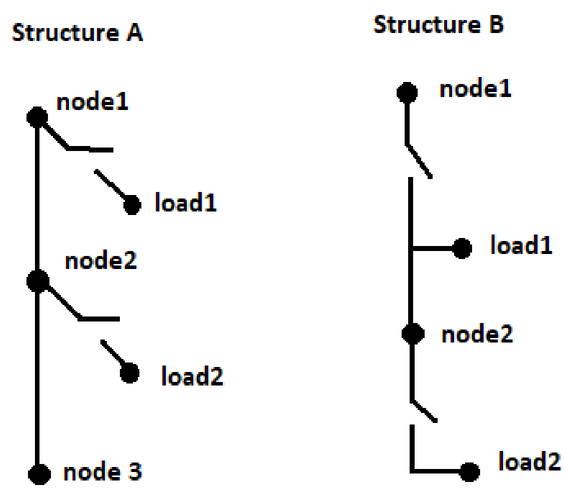

Figure 2.1: Two structures of distribution network

Let us start from the voltage constraints first.

2.1.1 *The Basic Power Flow Equations Model in Distribution Network*

In order to get the mathematical expression of node voltages, we need to begin with power flow equations. The radial distribution network used in this thesis is denoted as an undirected graph $\mathcal{G}$=($\mathcal{V}$,$\mathcal{E}$). $\mathcal{V}$ is the set of nodes including the source node/feeder N_0. $\mathcal{E}$ is the set of edges of the network. Figure 2.2 is a example of 13 nodes radial distribution network. In such a system, all the customers' demand are supplied by the only feeder of the system. There are no distributed generators (DGs) and compensators in the system. We assume that there are $n+1$ nodes in the

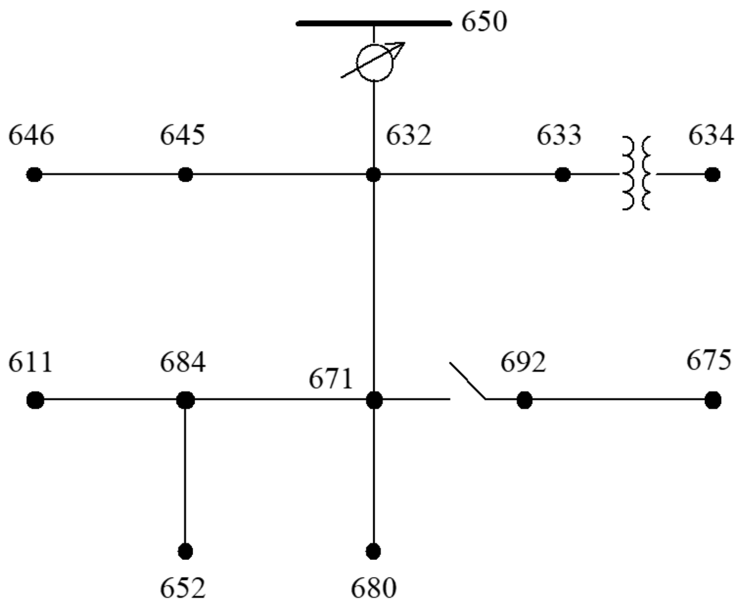

Figure 2.2: 13 nodes radial distribution network

network and we can get some basic power flow equations as follows:

$$S_{i,j} = V_i I_{i,j}^* \qquad \forall (i,j) \in \mathcal{E} \tag{1}$$

$$P_{i.j} - \sum_{l:(j,l)\in\varepsilon} P_{j.l} = p_j + r_{i,j}\frac{P_{i,j}^2 + Q_{i,j}^2}{|V_i|^2} \qquad \forall (i,j) \in \mathcal{E} \tag{2}$$

$$Q_{i.j} - \sum_{l:(j,l)\in\varepsilon} Q_{j.l} = q_j + x_{i,j}\frac{P_{i,j}^2 + Q_{i,j}^2}{|V_i|^2} \qquad \forall (i,j) \in \mathcal{E} \tag{3}$$

$$V_i - V_j = z_{i,j} I_{i,j} \qquad \forall (i,j) \in \mathcal{E} \tag{4}$$

$$|V_i|^2 - |V_j|^2 = 2(r_{i,j}P_{i,j} + x_{i,j}Q_{i,j}) - |I_{i,j}|^2|z_{i,j}|^2 \qquad \forall (i,j) \in \mathcal{E} \tag{5}$$

where $S_{i,j}$, $P_{i,j}$ and $Q_{i,j}$ are the apparent power, active power and reactive power transmitted from node i to node j. p_j and q_j represents the active load and the reactive load at node j. $r_{i,j}$, $x_{i,j}$ and $z_{i,j}$ are the resistance, the reactance and the impedance of line (i, j).

2.1.2 Linearization of Power Flow Model

We use DistFlow [5] equations to characterize the relationship between voltage and power flow. However, quadratic terms are included in three equations, which makes solving this problem more difficult. In order to get rid of the quadratic terms, we make two reasonable assumptions here.

- The loss on line is negligible
- The per unit value of node voltages are close to 1

Therefore we could neglect the quadratic terms in eqn. (3), (4) and (5). Also we could replace $|V_i|^2 - |V_j|^2$ by $2(|V_i| - |V_j|)$ [59]. As a result, eqn. (3), (4) and (5) can be simplified as:

$$P_{i.j} - \sum_{l:(j,l)\in\epsilon} P_{j.l} = p_j \qquad \forall (i,j) \in \mathcal{E} \tag{6}$$

$$Q_{i.j} - \sum_{l:(j,l)\in\epsilon} Q_{j.l} = q_j \qquad \forall (i,j) \in \mathcal{E} \tag{7}$$

$$|V_i| - |V_j| = r_{i,j}P_{i,j} + x_{i,j}Q_{i,j} \qquad \forall (i,j) \in \mathcal{E} \tag{8}$$

Eqn. (8) shows the linear relationship between node voltages and the power flow on the corresponding line. However, the power flow on each line are unknown to us during the solving process. So it is necessary to find out a way to denote the node voltages with some given variables. As shown in [25], we could use the load at each node to express the node voltage. And eqn. (8) can be written in matrix form:

$$\mathbf{V} - \mathbf{V_0} = \mathbf{RP} + \mathbf{XQ} \tag{9}$$

$\mathbf{V} \in \mathbb{R}^n$ is the column vector of node voltage magnitude $[|V_1|, \cdots, |V_n|]^T$. $\mathbf{V_0} \in \mathbb{R}^n$ is the column vector consisting of duplicated voltage magnitude of feeder node $[|V_0|, \cdots, |V_0|]^T$. $\mathbf{P}$ and $\mathbf{Q}$ represent the column vectors of injection power at each node, $[P_1, \cdots, P_n]^T$ and $[Q_1, \cdots, Q_n]^T$, which are the opposite number of load at each node. $\mathbf{R}$, $\mathbf{X} \in \mathbb{R}^{n\times n}$ are matrices and their elements

R_{ij} and X_{ij} are shown as follows:

$$R_{ij} = \sum_{\varepsilon' \in \mathcal{R}_i \cap \mathcal{R}_j} r_{\varepsilon'} \tag{10}$$

$$X_{ij} = \sum_{\varepsilon' \in \mathcal{R}_i \cap \mathcal{R}_j} x_{\varepsilon'} \tag{11}$$

where $\mathcal{R}_i$ is the route from node 0 to node i. R_{ij} and X_{ij} are the resistance and reactance of the overlapping route from feeder to these two nodes. With eqn. (9), we are able to denote voltage constraints in our optimization problem.

2.1.3 Multidimensional Knapsack Problem

The goal of the formulation process is to convert the load pickup problem into a multidimensional knapsack (d-KP) problem. The knapsack problem is one kind of decision problem, which is defined as follows: We are given an item set N with n items and each item has a value of v_j and a weight of w_j. The objective is to obtain an optimal subset of N with maximum profit and the total weight does not exceed the capacity limit c [20]. The d-KP problems is a special form with more than one capacity constraint. In distribution network, whether the loads are picked up or not are decided by a single switch connected to the bus. So we could aggregate all the loads connected to one bus as one large load, which means each node only need one decision variable.

First we need to quantify the load value v, which is illustrated as follows:

$$v_i = w_i p_i \quad \forall i \in \mathcal{V}/N_0 \tag{12}$$

where p_i is the active load at node i. Since only the active load has economic value, the reactive part is not included in the definition. Then we need to introduce a decision variable for each load. The binary decision variable y_i indicates the pickup status of each node, that $y_i = 1$ means the load at node i is picked up whereas $y_i = 0$ shows the opposite decision. So far we have obtained the optimization problem:

$$\max_{\{y_i\}} \sum_{i=1}^{n} w_i p_i y_i \tag{13}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} y_i p_i \leq P_{max} \tag{14}$$

$$\sum_{i=1}^{n} y_i q_i \leq Q_{max} \tag{15}$$

$$V_{min} \leq |V_i| \leq V_{max} \quad i = 1, \cdots, n \tag{16}$$

$$y_i \in \{0, 1\} \quad i = 1, \cdots, n \tag{17}$$

P_{max} and Q_{max} are the capacity limit of source generator/feeder. Substitute the node voltage with eqn. (9) and a new expression of constraint. (16) is shown:

$$|V_0| - V_{max} \leq \sum_{j=1}^{n} R_{ij} y_j p_j + \sum_{j=1}^{n} X_{ij} y_j q_j \leq |V_0| - V_{min} \quad i = 1, \cdots, n. \tag{18}$$

But constraint. (18) is still not a standard constraint of knapsack problem since it has both lower bound and upper bound. The standard constraints of knapsack problem have only upper bound. In order to eliminate the lower bound, we make an assumption that the voltage is decreasing along each branch and some researchers have illustrated that this assumption is reasonable. In [40], the APPENDIX-II shows us that $\frac{r}{x} \geq |\frac{q}{p}|$ is true in most of the cases. What's more is that most of the loads used in [18] and [52] are positive, which means that $\sum_{j=1}^{n} R_{ij} y_j p_j + \sum_{j=1}^{n} X_{ij} y_j q_j$ is positive. In [45], it gives out a data sheet of a 33-bus radial disribution system. It is pretty obvious that the bus voltage is decreasing along distribution line. In [33], the IEEE 33-bus distribution grid model is also used for analysis. And we can also find out the voltage is decreasing along the distribution line. [46] and [13] used the same result for analysis and all leads to the conclusion that voltage tends to decrease along the distribution line. Based on this assumption, it is reasonable to neglect the lower bound of constraint. (18). Finally we get the multidimensional knapsack problem we need:

$$\max_{\{y_i\}} \sum_{i=1}^{n} w_i p_i y_i \tag{19}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} y_i p_i \leq P_{max} \tag{20}$$

$$\sum_{i=1}^{n} y_i q_i \leq Q_{max} \tag{21}$$

$$\sum_{j=1}^{n} y_j (R_{ij} p_j + X_{ij} q_j) \leq |V_0| - V_{min} \quad i = 1, \cdots, n. \tag{22}$$

$$y_i \in \{0, 1\} \quad i = 1, \cdots, n \tag{23}$$

However, the formulation above only works for structure A. If the structure of the distribution network is type B, one more constraint is crucial:

$$y_i \geq y_j \quad i \in \mathcal{R}_j \tag{24}$$

which indicates that the load downstream could only be picked up if the load at its parent node is picked up.

2.2 Approximation Algorithms

The $n + 2$-dimensional knapsack problem above could be solved as an ILP problem. We could get the solution easily with appropriate solver. But there is one concern we need to pay attention, the computing time. D-KP problems are NP-hard problems [26] and usually cost much more time than linear programming (LP) problems. So it is necessary to find out an approximation algorithm with good performance and less computing time. However, it is unfortunate that fully polynomial time approximation schemes (FPTAS) does not exist [20]. So the only hope is polynomial time approximation schemes(PTAS). Some PTAS are proposed in [6], [7] and [15]. Even though PTAS have the drawback that the running time increases exponentially due to accuracy, they still perform much better than ILP. We will use $H^{1/(d+1)}$ algorithm [20] as our approximation algorithm, a PTAS based on the result from linear programming relaxation. And this algorithm is proved to be a $1/(d + 1)$-approximation algorithm.

2.2.1 *Linear Programming Relaxation*

In order to apply the approximation algorithm, linear programming relaxation (LKP) is indispensable. The LKP of the original problem could be derived by removing the integer constraint of each variable, thus getting the problem

$$\max_{\{y_i\}} \sum_{i=1}^{n} w_i p_i y_i \tag{25}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} y_i p_i \leq P_{max} \tag{26}$$

$$\sum_{i=1}^{n} y_i q_i \leq Q_{max} \tag{27}$$

$$\sum_{i=1}^{n} y_i (R_{ji} p_i + X_{ji} q_i) \leq |V_0| - V_{min} \qquad j = 1, \cdots, n. \tag{28}$$

$$0 \leq y_i \leq 1 \qquad i = 1, \cdots, n \tag{29}$$

We denote the objective value in the optimal solution of the original ILP problem as z^* and that one of LKP as z^{LP}. Obviously we can get

$$z^* \leq z^{LP} \tag{30}$$

2.2.2 $H^{1/(d+1)}$ *Approximation Algorithm*

The detail of the algorithm is described as follows:

Algorithm 1: $H^{1/(d+1)}$

Compute an optimal basic solution of decision variables y^{LP} of the LKP of d-KP

y^{LP} has no more than d fractional values where d is the number of constraints;

I : = $\left\{i | y_i^{LP} = 1\right\}$ variables with integer values;

F : = $\left\{i | 0 < y_i^{LP} < 1\right\}$ variables with fractional values;

z^H : = max$\left\{\sum_{i \in I} v_i, \max\left\{v_i | i \in \mathbf{F}\right\}\right\}$;

z^H is the solution of the $H^{1/(d+1)}$ approximation algorithm. The further detail of this algorithm is discussed in [14] and [7]. It is not difficult to draw out the conclusion that $H^{1/(d+1)}$ is a $1/(d+1)$-approximation algorithm, since

$$z^* \leq z^{LP} \leq \sum_{i \in \mathbf{I}} v_i + d\, F_{max} \leq (d+1) z^H \tag{31}$$

where F_{max} = max$\{v_i \mid \text{i} \in \mathbf{F}\}$.

As the original ILP problem we get is a $n+2$-KP problem, the algorithm would be a $1/(n+3)$-approximation algorithm. If the algorithm is applied to a large scale system, the value of n would be very large. In this case, the performance of our approximation algorithm seems to be unacceptable. However, d is actually the number of binding constraints. And based on our assumption of node voltage, we could only keep the voltage constraints of the leaf nodes, which means many constraints could be cancelled. Thus, the number of binding constraints decreases dramatically. So the performance of the approximation algorithm is not that bad.

2.3 Case Studies

We apply the knapsack ILP algorithm and $H^1/(d+1)$ approximation algorithm to 38, 123 and 8500 nodes test systems to examine the accuracy of the approximation algorithm and the computing time with different load settings. The Gurobi 8.1 solver on a 3.1 GHz Intel x64-based processor with 16B of RAM in a Julia 1.0.2 environment is used to solve the problem.

2.3.1 38-Node Test System

We first test ILP algorithm and $H^{1/(d+1)}$ algorithm on a 38-node test system. The system is shown as Figure 2.3. Node 1 is the feeder and no load is connected to it. So there are 37 loads in this system and we use random number to set the active load, reactive load and weight at each node. We first tried 10 different random seeds and compared the performance of these two algorithms. According to the result of ten times tests shown in Table 2.1, we find out that the objective value in the solution of $H^{1/(d+1)}$ algorithm is less than but very close to that of ILP and perform better

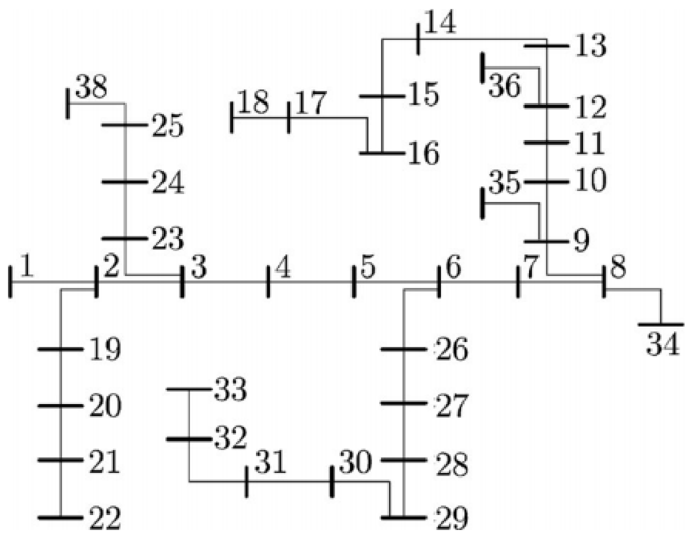

Figure 2.3: 38-node radial distribution network

Table 2.1: Performance of two algorithms on 38-node system

Case	Objective value of ILP	Objective value of $H^{1/(d+1)}$	Computing time of ILP/s	Computing time of $H^{1/(d+1)}$/s
1	31.3937	27.0112	0.0323	0.0063
2	38.8709	35.7887	0.0087	0.0058
3	50.1112	49.1009	0.0083	0.0061
4	46.5120	46.0184	0.0214	0.0058
5	41.0864	35.6971	0.0156	0.0145
6	36.3676	32.6767	0.0109	0.0058
7	46.8337	44.7128	0.0109	0.0060
8	48.2205	47.7839	0.0075	0.0059
9	41.7107	34.8491	0.0229	0.0055
10	37.9873	35.3640	0.0229	0.0060

in computing time. Then we tried 10000 times and the comparison results are illustrated in Figure 2.4 and Figure 2.5. Both figures are in histogram format and the Y-axis is number of cases

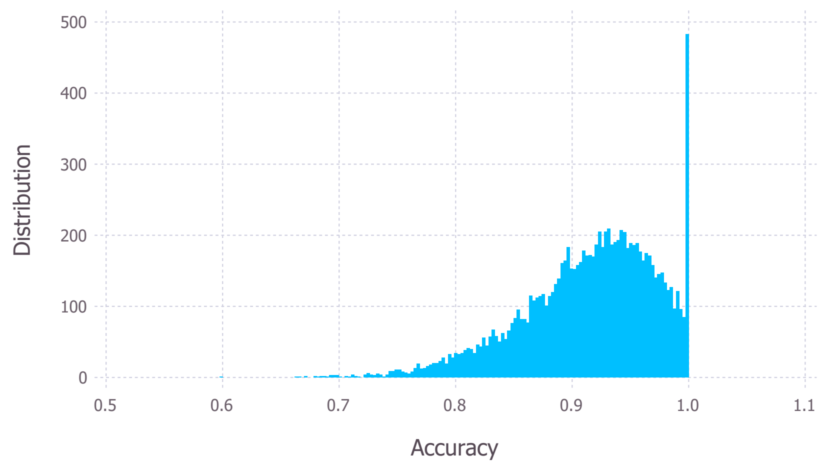

Figure 2.4: The accuracy distribution of 38-node system

among the 10000 tests corresponding to each X-axis value. The X-axis of Figure 2.4 represents the quotient of the objective value of two algorithms and the X-axis of Figure 2.5 represents the ratio of the computing time of two algorithms. The 10000 times results illustrate that the accuracy of the approximation algorithm is very good. However, in some cases the computing time of $H^{1/(d+1)}$ algorithm is longer than that of ILP. Another issue is that the longest computing time of ILP algorithm among the 10000 times test is only 0.1720s. This means that the ILP algorithm could get the accurate solution very fast and the application of approximation algorithm in this case is trivial.

2.3.2 123-Node Test System

Next step we applied our algorithms to IEEE 123-node test system in order to find out whether a more complex system will make the performance of approximation algorithm better. The distribution network is shown in Figure 2.6. Node 150 is the feeder. Switch (150, 149), (13, 152), (18,

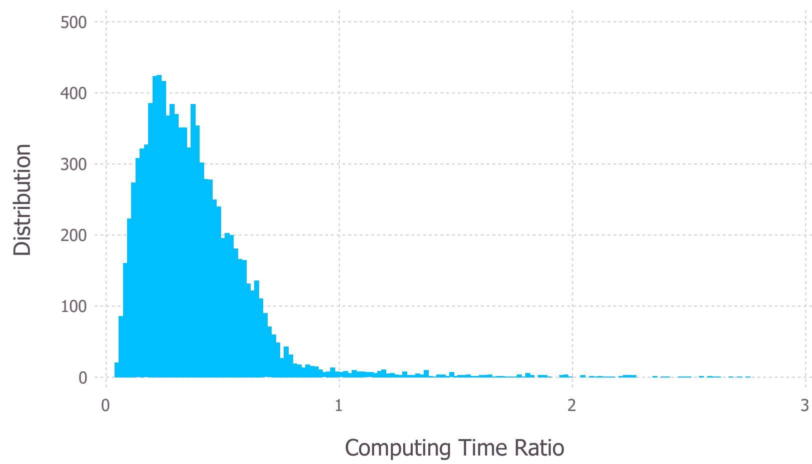

Figure 2.5: The computing time ratio distribution of 38-node system

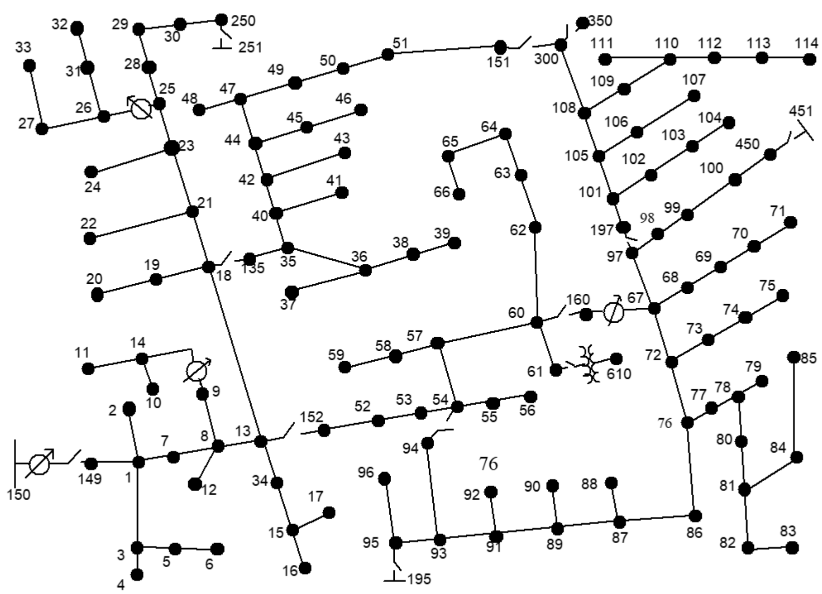

Figure 2.6: 123-node radial distribution network

135), (60, 160), (61, 610) and (97, 197) are closed in normal operation in order to keep the radial structure of the system. We compare the performance of two algorithms as the same process in 38-node system. Table 2.2 displays the ten times test results. We can see that the computing time

Table 2.2: Performance of two algorithms on 123-node system

Case	Objective value of ILP	Objective value of $H^{1/(d+1)}$	Computing time of ILP/s	Computing time of $H^{1/(d+1)}$/s
1	54.3326	43.4454	0.3353	0.0186
2	67.1716	59.0258	0.1455	0.0184
3	67.3135	61.1385	0.1422	0.0186
4	85.1487	75.4262	0.2267	0.0185
5	61.8342	47.0094	0.2849	0.0183
6	67.5476	55.0579	0.1699	0.0164
7	71.4997	67.8918	0.1737	0.0190
8	74.0108	59.2718	0.2699	0.0178
9	69.15745	61.4732	0.2534	0.0306
10	65.6802	52.8126	0.0517	0.0176

of $H^{1/(d+1)}$ algorithm is much shorter than that of ILP and the objective value is close. Also the 10000 times tests were tried out and the comparison results are shown in Figure 2.7 and Figure 2.8. As shown in the results, the approximation algorithm still has a good performance. But the main concern still is that the longest computing time of ILP is very short, which is only 0.5736s, making the approximation algorithm unnecessary.

2.3.3 8500-Node Test System

The IEEE 8500-node test system is used to test our algorithms for the final step. Since the accuracy has already been sufficiently tested in previous experiments, we will aim at the performance of computing time in this case. We will carry out experiments with different load settings to find out the computing time of ILP in the worst case. In other words, we are trying to find out the longest computing time of ILP algorithm. If the computing time of the worst case is sufficiently long, the approximation algorithm is still helpful. However, if the longest computing time is still within our

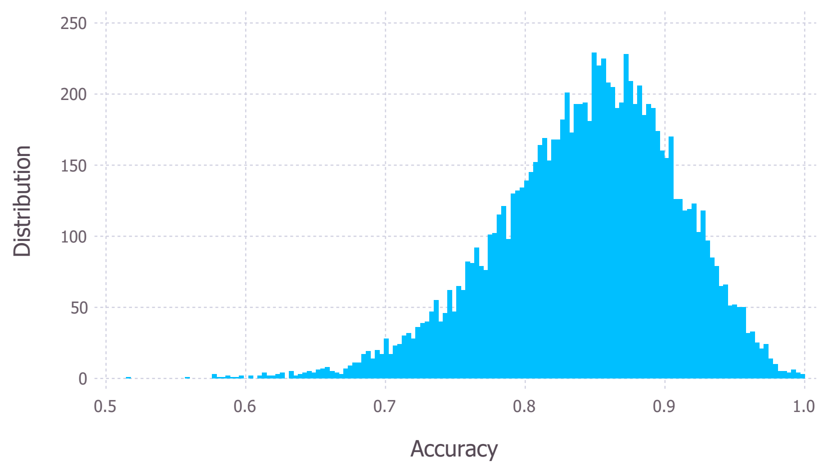

Figure 2.7: The accuracy distribution of 123-node system

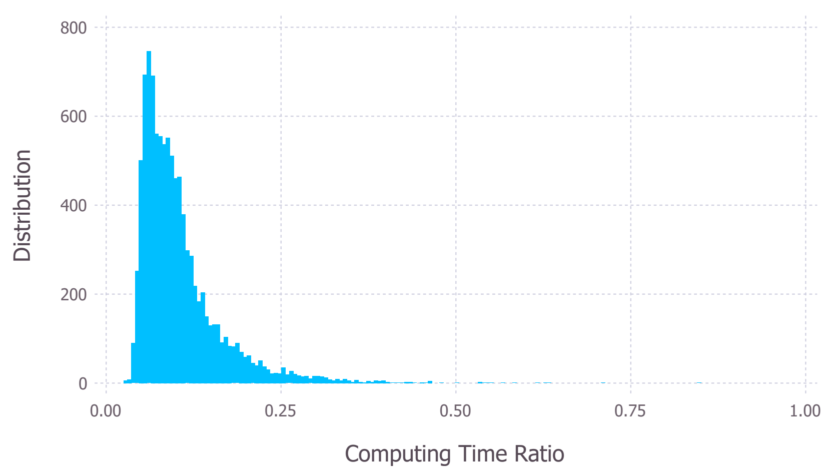

Figure 2.8: The computing time ratio distribution of 123-node system

worst expectation, the application of the approximation algorithm still needs further consideration. This time we consider both structures of distribution system as mentioned previously. We keep changing the load settings and the total active power constraints to find out the worst computing time of ILP algorithm. Figure 2.9 and Figure 2.10 depict our first attempt. We keep the same

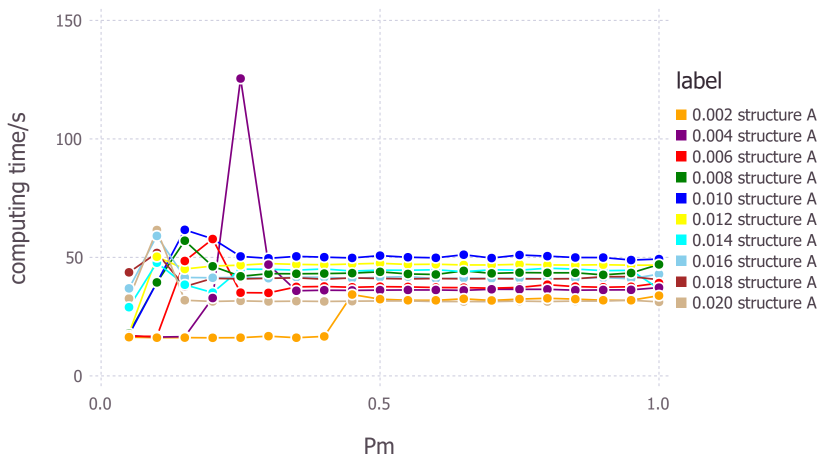

Figure 2.9: The computing time of ILP in an 8500-node system with structure A(0.002 0.020)

random seed for load generator and multiply the load values with different constants so that there are 10 different cases illustrated as the labels on the right side of each figure. The test is to find out the variation tendency of the ILP computing time when the active power constraint is increasing. The X-axis is the value of active power constraint, which is represented by the ratio of active power constraint and the sum of total loads. According to these two figures, we could find out that the longest computing time appears when the constant is very small in both network structures cases. Therefore, we replaced the constants we used in previous cases with smaller value and decreased the interval of each constant to find out the longest computing time in our cases. As a result, we obtained Figure 2.11 and Figure 2.12.

From the results, we obtain the longest computing time of ILP in our tests, which is 694.1637s. And we can get the computing time of our approximation algorithm in the same case, which is 252s. In conclusion, the $H^{1/(d+1)}$ algorithm does decrease the computing time obviously.

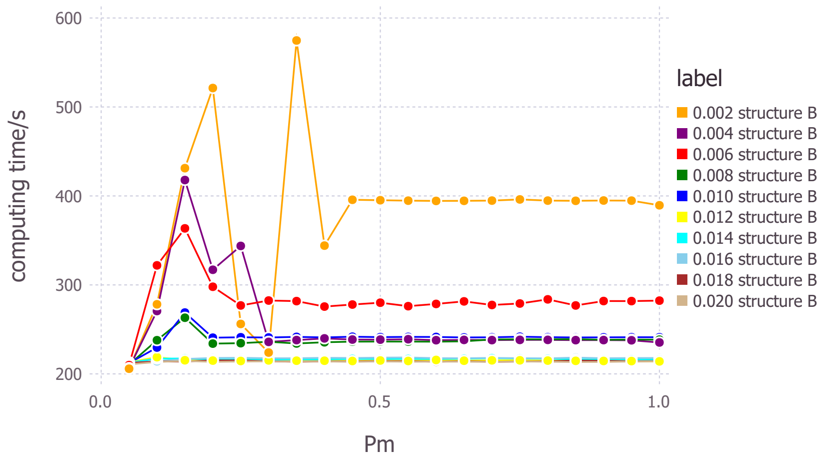

Figure 2.10: The computing time of ILP in an 8500-node system with structure B(0.002 0.020)

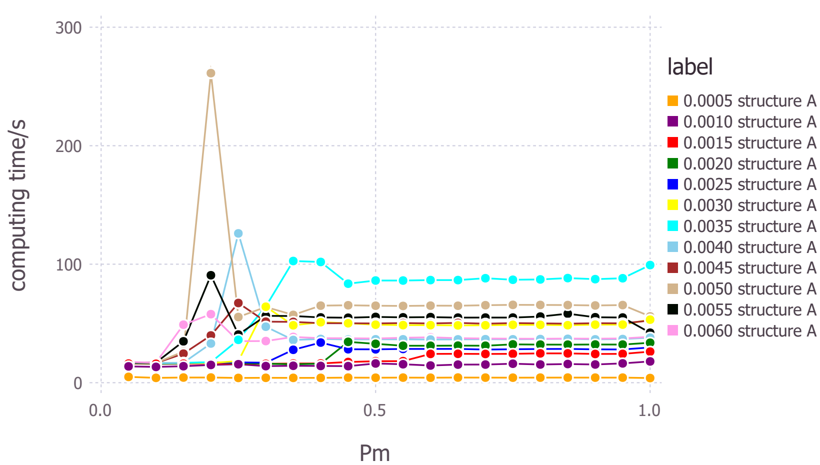

Figure 2.11: The computing time of ILP in an 8500-node system with structure A(0.0005-0.006)

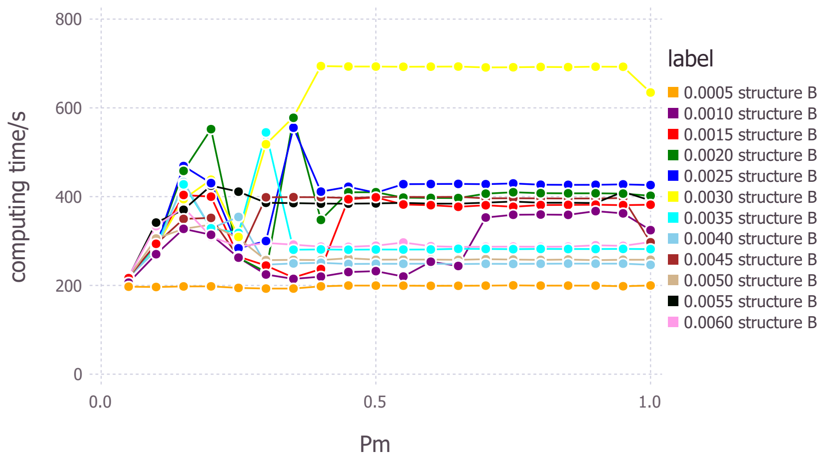

Figure 2.12: The computing time of ILP in an 8500-node system with structure B(0.0005-0.006)

2.4 Summary

Based on the results obtained from so many tests, we find out that the $H^{1/(d+1)}$ approximation algorithm could get a good solution close to the accurate one obtained by ILP algorithm with much less time. But the main problem is that the computing time of ILP is not very slow. In fact, the computing time 694s in the 8500-node test is somehow within our expectation. There are two possible reasons to explain why the performance of the ILP algorithm is better than our prediction. The first one is the presolve function of the Gurobi solver. The Gurobi solve could cancel some redundant constraints when it starts to solve the ILP problem. The second one is that the actual number of binding constraints is not very large, which enhances the solving speed of the solver. As a result, whether it is helpful to apply the approximation algorithm in our cases still needs further discussion. However, the load pickup problem could be upgraded to demand response problem [22] in some practical cases. The demand response problem could involve more customers' demands at one node. And even the elastic demand could be included, making the computing process amply complicated when solving the problem. The $H^{1/(d+1)}$ approximation algorithm would have a better performance then.

Chapter 3
FEEDER RECONFIGURATION

3.1 Problem Description

As stated in Secton 1.3, the goal of feeder reconfiguration is to find the best topological structure of the distribution system in order to decrease the power loss or to enhance the system performance. We aim to solve the load shifting problem during the fault period using feeder reconfiguration in this thesis. In a distribution system, there are several feeders and each of them has a radial network connected to them. If a fault happens to one of the feeders causing the capacity decrease, then the system operators need to shift some of the loads connected to the faulty feeder previously to other feeders. In this process, we still cannot the break the voltage constraints, line capacity constraints or feeder capacity constraints. Also the system after the reconfiguration process still need to obey the radiality constraints, which means the system connected to each of the feeders are still in radial structure. We are trying to come up with a mathematical method to solve the feeder reconfiguration problem in this thesis and the formulation of load pickup problem has given us an intuition of using ILP algorithm to solve the reconfiguration problem. The experiment result of load pickup problem shows that Gurobi solver could solve ILP problem pretty fast. Therefore the approximation algorithm is not necessary in this case. The systems used in this chapter are still single-phase system and some of the system data used in tests is modified from that of three-phase systems.

There are several problems when we are formulating the feeder reconfiguration problem.

- How to define our objective? Our purpose is to keep all the loads connected to the system during the reconfiguration process. Meanwhile, we would like to shift the loads with a minimum number of switch operation. So it is essential to come up with a way to quantify this objective. For convenience, we assume that all the loads could be connected to feeders after the reconfiguration, which means there is no load shedding.

- How to use the DistFlow equations in this problem? As we know, in order to correlate the node voltages with the power flow, the DistFlow equation would be a appropriate tool. However, various from the load pickup problem, the topological structure of the distribution system changes during the reconfiguration process. As a result, it is impossible to use the DistFlow equations directly unless we know the system structure in the solution.

- How to attach the radiality constraints to the optimization problem? A method to realize the function of the radiality constraints with a minimum number of formulas is fundamental to the formulation, since adding too many constraints would reduce the efficiency of the solver

3.2 Problem Formulation of 16-node system

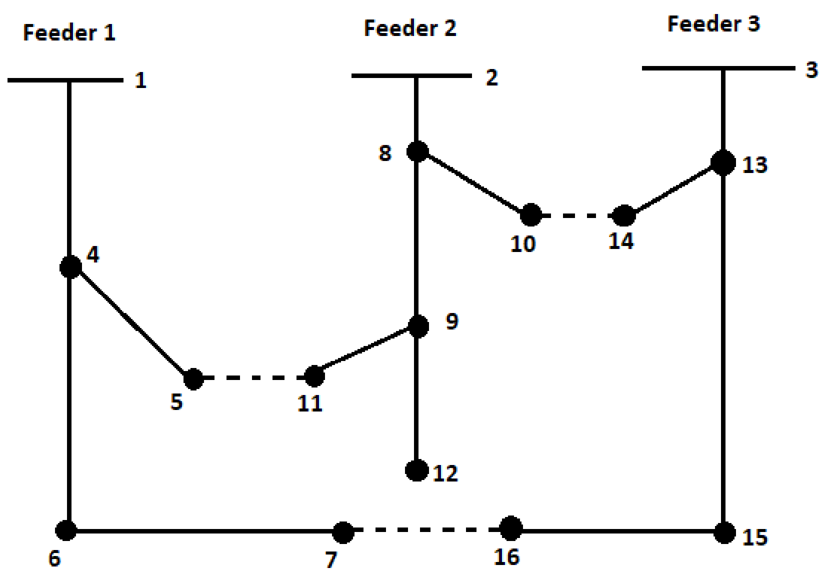

Figure 3.1: 3-feeder 16-node distribution network

Let us start from the classical three-feeder 16-node distribution network model [10]. The data is shown in Table 3.1. The apparent power base is 100 MVA and the voltage base is 23 kV in this case. There are three feeders and three independent radial subsystems in this distribution system.

Table 3.1: Data of the three-feeder 16-node system

Bus to Bus	Resistance/P.U	Reactance/P.U	End Bus Load/MW	End Bus Load/MVAR
1-4	0.075	0.1	2.0	1.6
4-5	0.08	0.11	3.0	1.5
4-6	0.09	0.18	2.0	0.8
6-7	0.04	0.04	1.5	1.2
2-8	0.11	0.11	4.0	2.7
8-9	0.08	0.11	5.0	3.0
8-10	0.11	0.11	1.0	0.9
9-11	0.11	0.11	0.6	0.1
9-12	0.08	0.11	4.5	2.0
3-13	0.11	0.11	1.0	0.9
13-14	0.09	0.12	1.0	0.7
13-15	0.08	0.11	1.0	0.9
15-16	0.04	0.04	2.1	1.0
5-11	0.04	0.04	-	-
10-14	0.04	0.04	-	-
7-16	0.09	0.12	-	-

And we assume that each bold line has a sectionalizing switch which is normally closed and each dashed line has a tie switch which is normally open. There are 16 switches in total. And 13 of them are sectionalizing switches and 3 of them are tie switches. The fault is set at Feeder 1, causing the capacity of Feeder 1 decrease. In order to eliminate the load shedding on Feeder 1, we need to shift some of the loads connected to Feeder 1 to another two feeders.

3.2.1 Objective Function

We are going to use ILP to solve the reconfiguration problem. Therefore, the first step is to define the objective function in the optimization problem. Two binary variables, y_i and z_j are used to indicate the status of the closed switches and the tie switches [58].

$$y_i = \begin{cases} 1, \text{closed switch i is closed.} \\ 0, \text{closed switch i is open.} \end{cases} \qquad i = 1, 2, ..., 13 \tag{1}$$

$$z_j = \begin{cases} 1, \text{tie switch j is closed.} \\ 0, \text{tie switch j is open.} \end{cases} \qquad j = 1, 2, 3 \tag{2}$$

Our objective is to keep all the loads connected to the system with the minimum number of switching operations. So we can get the objective function:

$$\min_{\{y_i, z_j\}} \sum_{i=1}^{13} (1 - y_i) + \sum_{j=1}^{3} z_j \tag{3}$$

Thus, we are able to express our objective with mathematical formula.

3.2.2 Line Capacity Constraints

The power flow on each line cannot exceed its own capacity. Normally, the line capacity is denoted by current. But in order to use ILP, we assume that the line capacity is characterized by the active power and the reactive power. The line capacity constraints are illustrated as follows:

$$-\omega_{ij} P_{cap} \leq P_{i,j} \leq \omega_{ij} P_{cap} \quad \forall (i, j) \in \mathcal{E} \tag{4}$$

$$-\omega_{ij} Q_{cap} \leq Q_{i,j} \leq \omega_{ij} Q_{cap} \quad \forall (i, j) \in \mathcal{E} \tag{5}$$

$\mathcal{E}$ is the set of all the lines in the system, including the disconnected lines with tie switches. P_{ij} and Q_{ij} are the active power and the reactive power on line (i, j). P_{cap} and Q_{cap} represent the line capacity. ω_{ij} indicates the connecting status of line (i, j).

Let

$$\omega_{ij} = \begin{cases} 1, & \text{line } (i,j) \text{ is connected.} \\ 0, & \text{line } (i,j) \text{ is disconnected.} \end{cases} \tag{6}$$

By introducing the binary variable ω, we are able to enforce that the power flow on a disconnected line is zero. Also we assume that the capacity of each line does not vary too much from each other[1]. So it is credible to propose the assumption that all the lines in the distribution network share the same capacity.

3.2.3 Feeder Capacity Constraint

$$P_k \leq P_{mk} \quad \forall k \in \mathcal{V}_F \tag{7}$$

$$Q_k \leq Q_{mk} \quad \forall k \in \mathcal{V}_F \tag{8}$$

The total loads P_k and Q_k connected to each feeder should be no more than its capacity limit P_{mk} and Q_{mk}. $\mathcal{V}_F$ is the set of feeders in the system. In normal cases, the capacity of feeder is denoted by apparent power. But using apparent power would involves quadratic terms in our optimization problem. So we divide the feeder capacity into two terms, the active power capacity and the reactive power capacity.

3.2.4 Voltage Magnitude Constraint

$$V_{min} \leq V_i \leq V_{max} \quad \forall i \in \mathcal{V}_N \tag{9}$$

$\mathcal{V}_N$ represents the set of all the nodes, including the feeders and the buses. Since we assume that there is no load shedding in the system, so all the node voltages should be restricted to this range. For convenience, we assume the voltage of the feeder is V_{max}.

[1]This will be further proved in the 375-node system in section 3.4.

3.2.5 DistFlow Equations using Big M Approach

Due to the topological structure change of the distribution network, using the DistFlow equations as what we have done in the load pickup problem is impossible. So we need to think of a new way to implement the DistFlow equations. [30] gives us a hint of using big M method. Then power flow constraints can be formulated as follows:

$$\sum_{j|(i.j)\in\mathcal{E}} P_{i,j} = -p_i \quad \forall i \in \mathcal{V}_B \tag{10}$$

$$\sum_{j|(i.j)\in\mathcal{E}} Q_{i,j} = -q_i \quad \forall i \in \mathcal{V}_B \tag{11}$$

$$V_i - (r_{i,j}P_{i,j} + x_{i,j}Q_{i,j}) - (1-\omega_{ij})M \leq V_j \leq V_i - (r_{i,j}P_{ij} + x_{i,j}Q_{i,j}) + (1-\omega_{ij})M \quad \forall (i,j) \in \mathcal{E} \tag{12}$$

Constraint. (10)- (11) are the power balance equations at each bus. $\mathcal{V}_B$ is the set of buses in the system. We neglect the power loss on the lines in the reconfiguration problem. The big M approach is used in constraint. (12). By introducing a sufficiently large constant M, the previous equality constraint is converted into an inequality constraint. If the two nodes i and j are connected, which means ω_{ij} is 1, constraint. (12) would be a normal DistFlow equation in this case. If these two nodes are not connected, the sufficiently large constant M would also keep this constraint satisfied. In this way, we could still use the DistFlow equations to correlate the node voltage with power flow.

3.2.6 Radiality Constraint

After we manage to implement the DistFlow equations, only one difficulty is left. It is essential to keep the radiality structure of the system after the reconfiguration process, which means that there are still three independent radial distribution networks connecting to each feeder and no loop circuit exists in the system. As we know that a necessary condition for network radiality is

$$\sum_{(i,j)\in\mathcal{E}} \omega_{ij} = N_n - N_f \tag{13}$$

N_n and N_f are the numbers of total nodes and feeders. But this condition is not enough, since loop circuit may appear in the system without breaking this constraint. We introduced two new binary variables β_{ij} and β_{ji}, corresponding to each line, to construct the radiality constraint [17]. The radiality constraints are

$$\beta_{ij} + \beta_{ji} = \omega_{ij} \quad \forall (i,j) \in \mathcal{E} \tag{14}$$

$$\beta_{kj} = 0 \qquad \forall k \in \mathcal{V}_F \tag{15}$$

$$\sum_{(i,j)\in\mathcal{E}} \beta_{ij} = 1 \qquad \forall i \in \mathcal{V}_B \tag{16}$$

$$\beta_{ij} \in \{0,1\} \tag{17}$$

β_{ij} indicates that node j is the parent of node i when its value is 1. Constraint. (14) shows that node i and node j are connected to each other if and only if line (i, j) is connected. Constraint. (15) means that the feeders have no parents. Constraint. (16) indicates that each bus has only one parent. Actually reference [3] shows that the constraints above still cannot guarantee the radial structure of the distribution and gives out an approach based on planar-dual graph [53]. However, the radiality constraints constraint. (14)- (16) still work with the help of power flow equations. The infeasible network structure, such as the loop circuit and unreasonable load shedding, leads to infeasible power flow solution [23]. So we can still get the right solution with the radiality constraints above.

3.3 Case Studies of Three-feeder 16-node System

The Gurobi 8.1 solver on a 3.1 GHz Intel x64-based processor with 16B of RAM in a Julia 1.0.2 environment is used to solve the problem. In order to examine the performance of the algorithm, we have carried out many test cases on the three-feeder distribution system to test if all the constraints work as we design and if the algorithm would give us a feasible solution even in some complicated cases. We will only show some corner test cases. The fault is located at Feeder 1 in all the following cases.

Table 3.2: Settings of Case 1 for three-feeder networks

P_{cap}=0.2 Q_{cap}=0.11				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.02	0.016	P_{m1}=0.07225	Q_{m1}=0.04335
5	0.03	0.015		
6	0.02	0.008		
7	0.015	0.012		
8	0.04	0.027	P_{m2}=0.2718	Q_{m2}=0.1566
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.01	0.009	P_{m3}=0.0918	Q_{m3}=0.063
14	0.01	0.007		
15	0.01	0.009		
16	0.021	0.01		

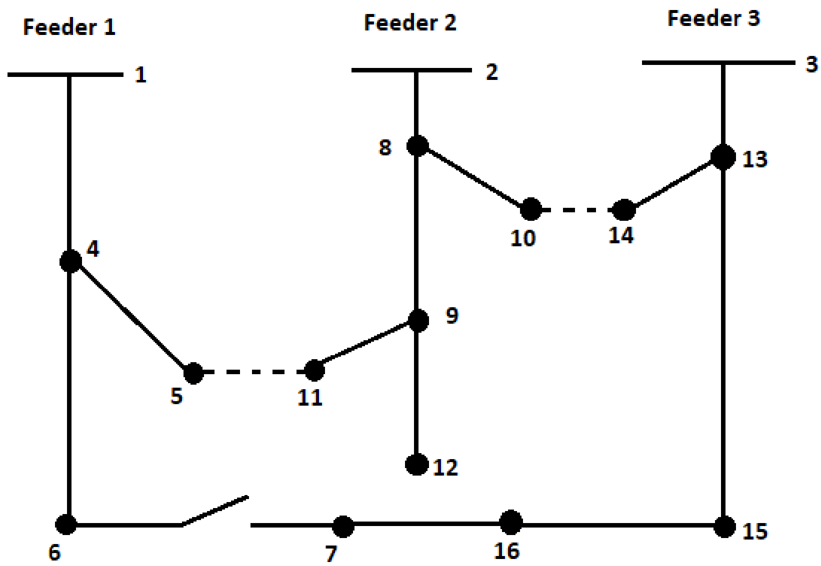

Figure 3.2: Solution of Case 1

3.3.1 Case 1

First, we carry out an easy case to make sure that the algorithm is working. Table 3.2 shows the setting of this case and Figure 3.2 illustrates the solution. It indicates that load 7 is shifted from Feeder 1 to Feeder 3. Switch (6, 7) is open and switch (7, 16) is closed. So the objective value is 2. Actually there is another feasible solution with an objective value of 2, in which load 5 is shifted from Feeder 1 to Feeder 2. The Gurobi solver could only gives out one of these feasible solutions.

3.3.2 Case 2

In order to examine if the feeder capacity constraint is working or not, we adjust the capacity of Feeder 3 in this case, making it smaller than that of case 2 shown as Table 3.3. And the solution is shown as Figure 3.3. In this case, we get the another solution we mentioned in case 1. Since shifting load 7 to Feeder 3 will break the capacity constraint of Feeder 3, there is only one feasible solution in this case and the objective value is 2. The feeder capacity of Feeder 3 is a binding

Table 3.3: Settings of Case 2 for three-feeder networks

P_{cap}=0.2 Q_{cap}=0.11				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.02	0.016	P_{m1}=0.07225	Q_{m1}=0.04335
5	0.03	0.015		
6	0.02	0.008		
7	0.015	0.012		
8	0.04	0.027	P_{m2}=0.2718	Q_{m2}=0.1566
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.01	0.009	P_{m3}=0.06	Q_{m3}=0.063
14	0.01	0.007		
15	0.01	0.009		
16	0.021	0.01		

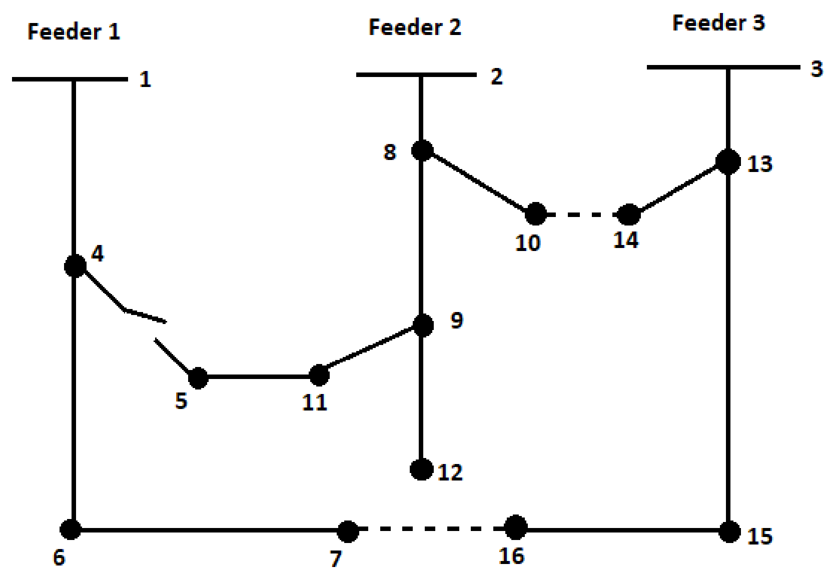

Figure 3.3: Solution of Case 2

constraint in this case.

3.3.3 Case 3

In this case, we try to assess the performance of the algorithm when the line capacity is a binding constraint. Table 3.4 gives out the settings. The only possible solution in this case is shifting load 4 and load 5 to Feeder 2 and shifting load 6 and load 7 to Feeder 3. However, doing this would break the line capacity constraint. So there is no feasible solution in this case. And the algorithm cannot give out a solution either in this case, which means the algorithm works well in this case.

3.3.4 Case 4

In this case, we try to find out if the voltage constraints are working or not. Table 3.5 shows the settings of this case. If we only consider the line capacity and the feeder capacity constraints, either shifting load 7 to Feeder 3 or shifting load 5 to Feeder 2 would work. But shifting load 7 will break the voltage constraint of node 7, which is a wrong solution. The solution given by the solver is the

Table 3.4: Settings of Case 3 for three-feeder networks

P_{cap}=0.2 Q_{cap}=0.11				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.02	0.016	P_{m1}=0.01	Q_{m1}=0.03
5	0.03	0.015		
6	0.02	0.008		
7	0.015	0.012		
8	0.04	0.027	P_{m2}=0.21	Q_{m2}=0.121
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.01	0.009	P_{m3}=0.087	Q_{m3}=0.056
14	0.01	0.007		
15	0.01	0.009		
16	0.021	0.01		

Table 3.5: Settings of Case 4 for three-feeder networks

P_{cap}=0.9 Q_{cap}=0.5				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.2	0.16	P_{m1}=0.71	Q_{m1}=0.5
5	0.3	0.15		
6	0.2	0.08		
7	0.15	0.12		
8	0.04	0.027	P_{m2}=0.5	Q_{m2}=0.3
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.1	0.09	P_{m3}=0.7	Q_{m3}=0.5
14	0.1	0.07		
15	0.1	0.09		
16	0.21	0.1		

same as Figure 3.3 and the objective value is 2 in this case.

3.3.5 Case 5

After making sure that all the constraints work well in the optimization problem, we would like to try out some complicated cases involving more switch operations. The setting of case is shown in Table 3.6. And the solution is given by Figure 3.4. In this case, load 5 is shifted from Feeder 1 to

Table 3.6: Settings of Case 5 for three-feeder networks

<table>
<tr><td colspan="5">P_{cap}=0.2 Q_{cap}=0.11</td></tr>
<tr><td>Node</td><td>P/P.U</td><td>Q/P.U</td><td>P_m/P.U</td><td>Q_m/P.U</td></tr>
<tr><td>4</td><td>0.02</td><td>0.016</td><td rowspan="4">P_{m1}=0.045</td><td rowspan="4">Q_{m1}=0.03</td></tr>
<tr><td>5</td><td>0.03</td><td>0.015</td></tr>
<tr><td>6</td><td>0.02</td><td>0.008</td></tr>
<tr><td>7</td><td>0.015</td><td>0.012</td></tr>
<tr><td>8</td><td>0.04</td><td>0.027</td><td rowspan="5">P_{m2}=0.19</td><td rowspan="5">Q_{m2}=0.105</td></tr>
<tr><td>9</td><td>0.05</td><td>0.03</td></tr>
<tr><td>10</td><td>0.01</td><td>0.009</td></tr>
<tr><td>11</td><td>0.006</td><td>0.001</td></tr>
<tr><td>12</td><td>0.045</td><td>0.02</td></tr>
<tr><td>13</td><td>0.01</td><td>0.009</td><td rowspan="4">P_{m3}=0.07</td><td rowspan="4">Q_{m3}=0.048</td></tr>
<tr><td>14</td><td>0.01</td><td>0.007</td></tr>
<tr><td>15</td><td>0.01</td><td>0.009</td></tr>
<tr><td>16</td><td>0.021</td><td>0.01</td></tr>
</table>

Feeder 2 and load 7 is shifted from Feeder 1 to Feeder 3. This is the only feasible solution for this case with an objective value of 4.

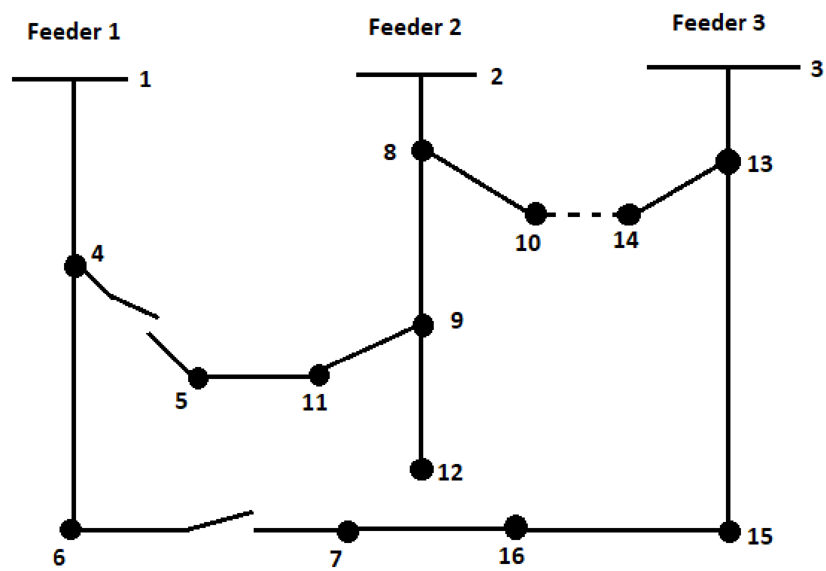

Figure 3.4: Solution of Case 5

3.3.6 Case 6

Table 3.7 shows the setting and the solution is illustrated by Figure 3.5. In this case, Feeder 1 has to shift all its load. As a result, load 4 and 5 are shifted to Feeder 2 and load 6 and 7 are shifted to Feeder 3. This is the only solution and the objective value is 4 in this case.

3.3.7 Case 7

This is the most complicated case of the three-feeder network with a maximum objective value. Table 3.8 displays the settings and the solution is shown as Figure 3.6. First, Feeder 1 tends to shift load at its leaf node. So load 7 is shifted to Feeder 3. But the total load connected to Feeder 1 still exceeds its capacity limit in this case. Meanwhile, shifting any more load from Feeder 1 to Feeder 3 would break the capacity constraints of Feeder 3. So Feeder 1 shifts load 5 to Feeder 2. However, this action breaks the capacity constraint of Feeder 2. Then Feeder 2 has to shift load 10 to Feeder 3. In the end, we get a feasible solution without breaking any constraints. The objective value is 6

Table 3.7: Settings of Case 6 for three-feeder networks

P_{cap}=0.21 Q_{cap}=0.15				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.02	0.016	P_{m1}=0.01	Q_{m1}=0.03
5	0.03	0.015		
6	0.02	0.008		
7	0.015	0.012		
8	0.04	0.027	P_{m2}=0.21	Q_{m2}=0.121
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.01	0.009	P_{m3}=0.087	Q_{m3}=0.056
14	0.01	0.007		
15	0.01	0.009		
16	0.021	0.01		

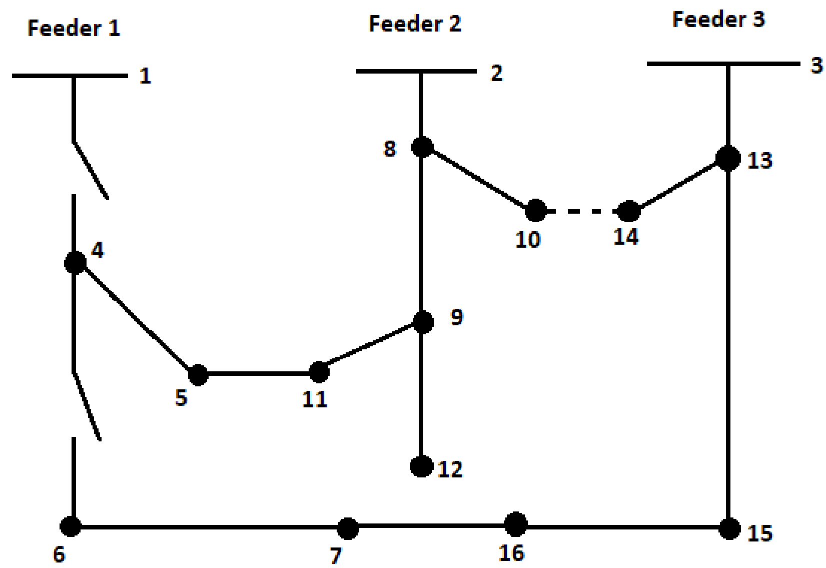

Figure 3.5: Solution of Case 6

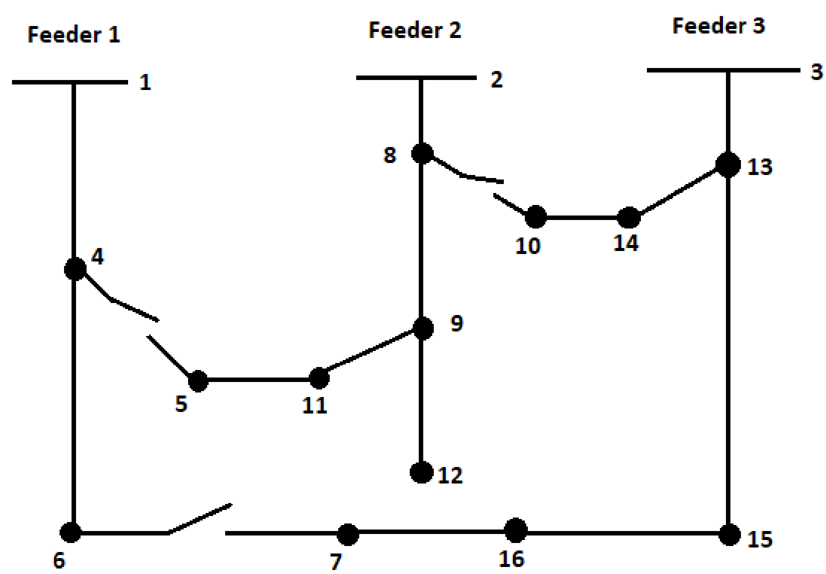

Figure 3.6: Solution of Case 7

Table 3.8: Settings of Case 7 for three-feeder networks

P_{cap}=0.2 Q_{cap}=0.11				
Node	P/P.U	Q/P.U	P_m/P.U	Q_m/P.U
4	0.02	0.016	P_{m1}=0.045	Q_{m1}=0.025
5	0.03	0.015		
6	0.02	0.008		
7	0.015	0.012		
8	0.04	0.027	P_{m2}=0.172	Q_{m2}=0.102
9	0.05	0.03		
10	0.01	0.009		
11	0.006	0.001		
12	0.045	0.02		
13	0.01	0.009	P_{m3}=0.08	Q_{m3}=0.057
14	0.01	0.007		
15	0.01	0.009		
16	0.021	0.01		

and this is the maximum objective value in the three-feeder 16-node distribution network.

3.4 Scale Up

In order to further examine the performance our algorithm, we need to apply it to a larger scale distribution system. We try to refer to some large scale distribution networks used by others. Debapriya [11] gives out the data of a four-feeder 70-node system and Ching-Tzong [49] gives out the data of an 11-feeder 83-node. The example of 415-node and 830-node system are proposed by Rabih [17], which are obtained by duplicating the 83 nodes system 5 times and 10 times. Dong [56] carries out his own test on an one-feeder 119-node system. The data of an eight-feeder 136-nodes system is shown in [31]. In the distribution networks above, it is assumed that each line has a switch which could isolate the faulted lines. However, it is not realistic to install switches on every line in a distribution network because of the high expense [1], [57]. Therefore we come up with a three-feeder 375-node distribution system, in which not all lines have switches. This distribution system consists of three identical independent radial distribution networks, each of which is modified from IEEE 123-node test system. Unlike the three-feeder 16-node system, we cannot disconnect a line without a switch attached. So some more constraints are essential, which are shown as follows:

$$1 - \omega_{ij} \leq sw_{ij} \tag{18}$$

$$sw_{ij} \in \{0, 1\} \tag{19}$$

The binary variable sw_{ij} indicates that the line (i, j) has a switch when its value is 1. With these constraints, only the lines with switches could be disconnected. The formulation of other constraints are similar to those of three-feeder 16-node case. The benefit of formulating in this way instead of aggregating the loads without switches into one big load is that we could impose the voltage constraint to each node and the line capacity constraint to each line.

About the line capacity assumption we have made in the 16-node system, we will show its rationality here. The line configuration data of the IEEE 123-node system is shown in Table 3.9. We can see in the table that the multi-phase lines share the same configuration and most of the

Table 3.9: Overhead Line Configurations of IEEE 123-node system

Config.	Phasing	Phase Cond.	Neutral Cond.	Spacing
		ACSR	ACSR	ID
1	ABCN	336,400 26/7	4/0 6/1	500
2	CABN	336,400 26/7	4/0 6/1	500
3	BCAN	336,400 26/7	4/0 6/1	500
4	CBAN	336,400 26/7	4/0 6/1	500
5	BACN	336,400 26/7	4/0 6/1	500
6	ACBN	336,400 26/7	4/0 6/1	500
7	ACN	336,400 26/7	4/0 6/1	505
8	ABN	336,400 26/7	4/0 6/1	505
9	AN	1/0	1/0	510
10	BN	1/0	1/0	510
11	CN	1/0	1/0	510

lines in the previous 123-node system are multi-phase lines. In addition, we modified the previous 123-node system into a single-phase system. Therefore we could assume that all the lines share the same line capacity.

3.5 Case Studies of Three-feeder 375-node System

We think of two possible structures of the 375-node distribution network, which are shown in Figure 3.7 and Figure 3.8. The sectionalizing switches and the tie switches are all shown in the figures and the tie switches are highlighted. There are four tie switches in the parallel structure system, in which only Feeder-II has connections with both Feeder-I and Feeder-III. There are six tie switches in the triangle structure system and any feeder has connections with another two feeders. We carry out many tests on both systems and even a special case where the system could change its internal structure for reconfiguration. The fault is located at Feeder-I in all the following test cases. The Gurobi 8.1 solver on a 3.1 GHz Intel x64-based processor with 16B of RAM in a Julia 1.0.2 environment is used to solve the problem.

3.5.1 Parallel Structure System

Case 1

The constraint settings are shown in Table 3.10. And the solution is shown as Figure 3.9. The

Table 3.10: Constraint Settings of Case 1 Parallel Structure System

P_{m1}/P.U	P_{m2}/P.U	P_{m3}/P.U	Q_{m1}/P.U	Q_{m2}/P.U	Q_{m3}/P.U	P_{cap}/P.U	Q_{cap}/P.U
6.432	9.648	9.648	3.616	5.424	5.424	10.67	6

switches (13-I, 18-I), (76-II, 77-II) and (97-II, 197-II) are open and tie switches (250-I, 250-II), (300-I, 300-II) and (83-II, 83-III) are closed. After accepting the load from Feeder-I, the total load connected to Feeder-II exceeds its capacity. So Feeder-II has to shift some of its load to another two

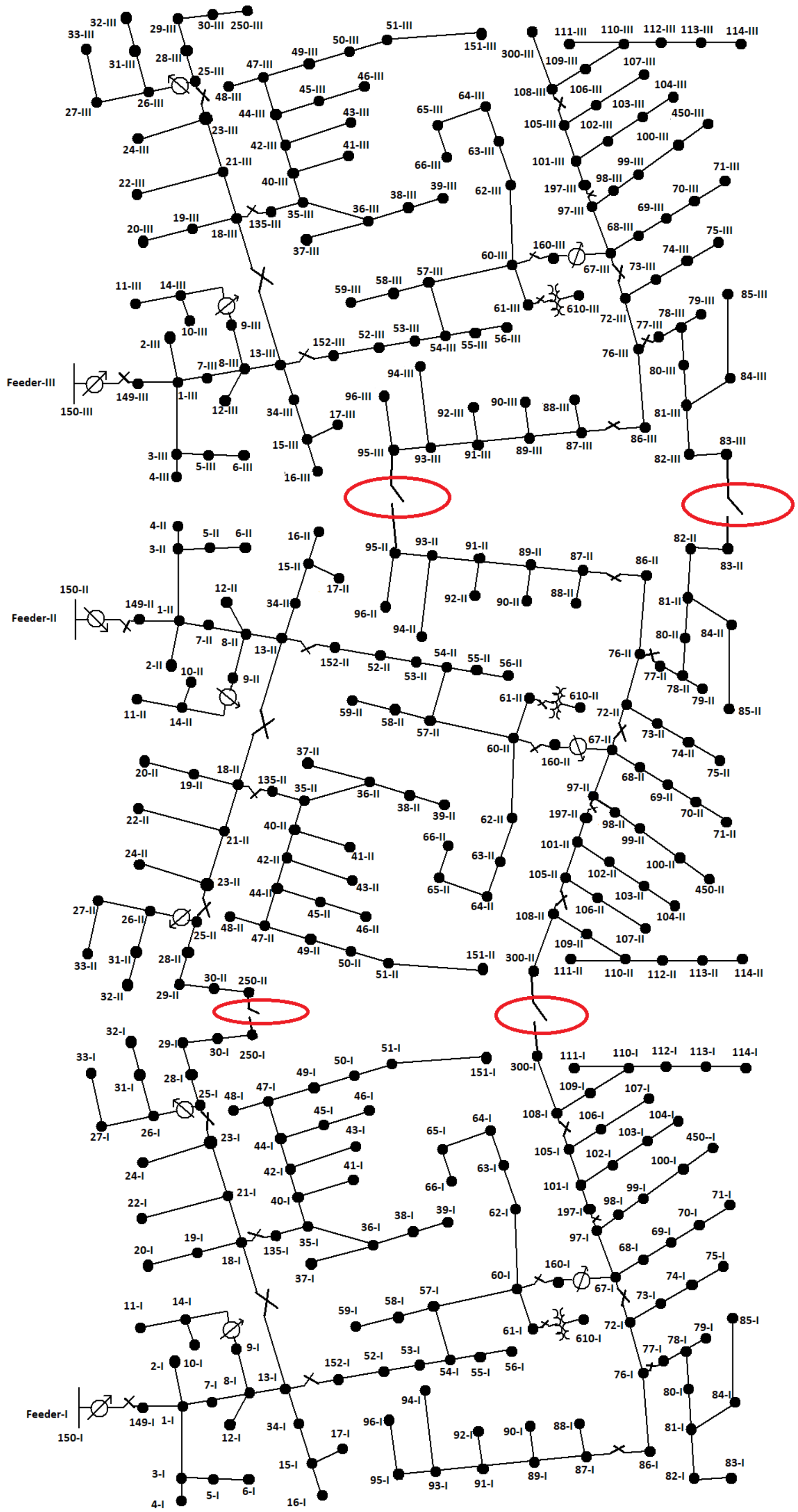

Figure 3.7: 375-node system in parallel structure

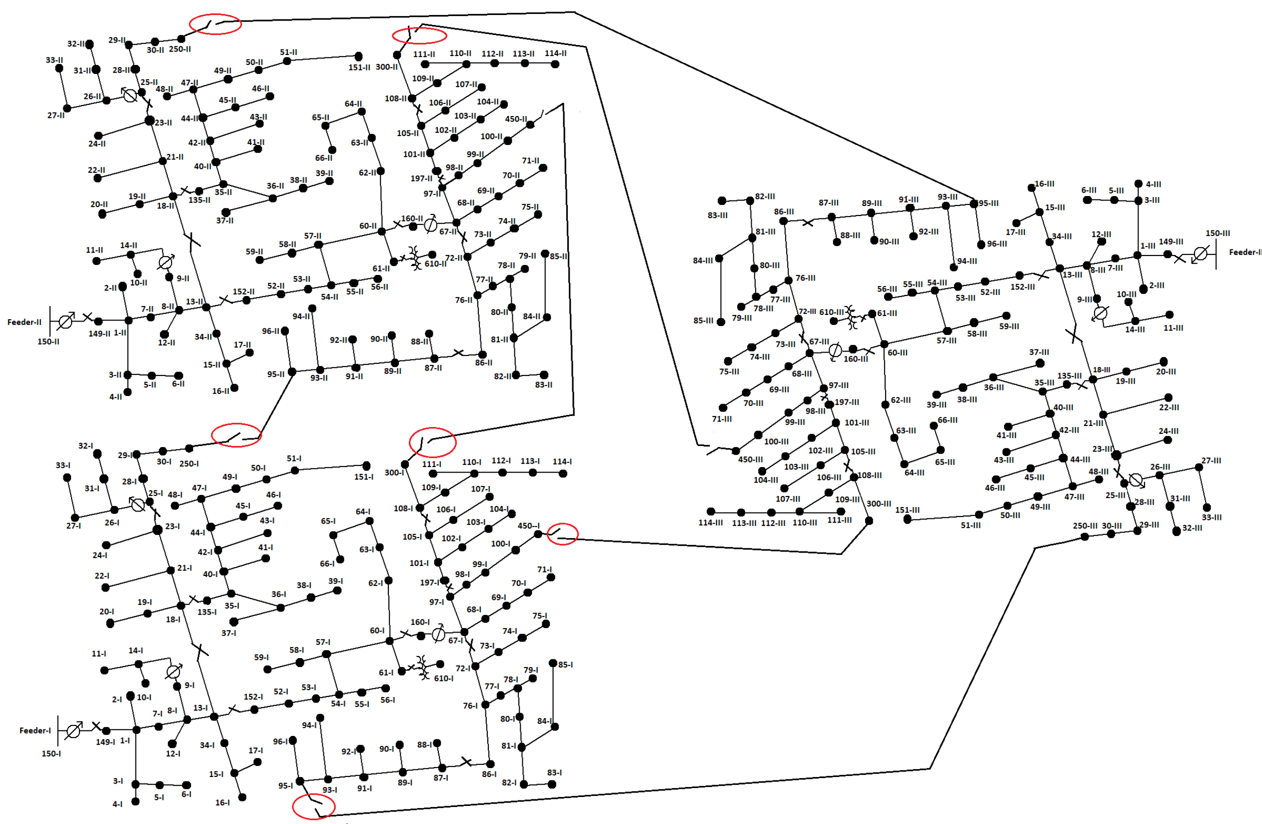

Figure 3.8: 375-node system in triangle structure

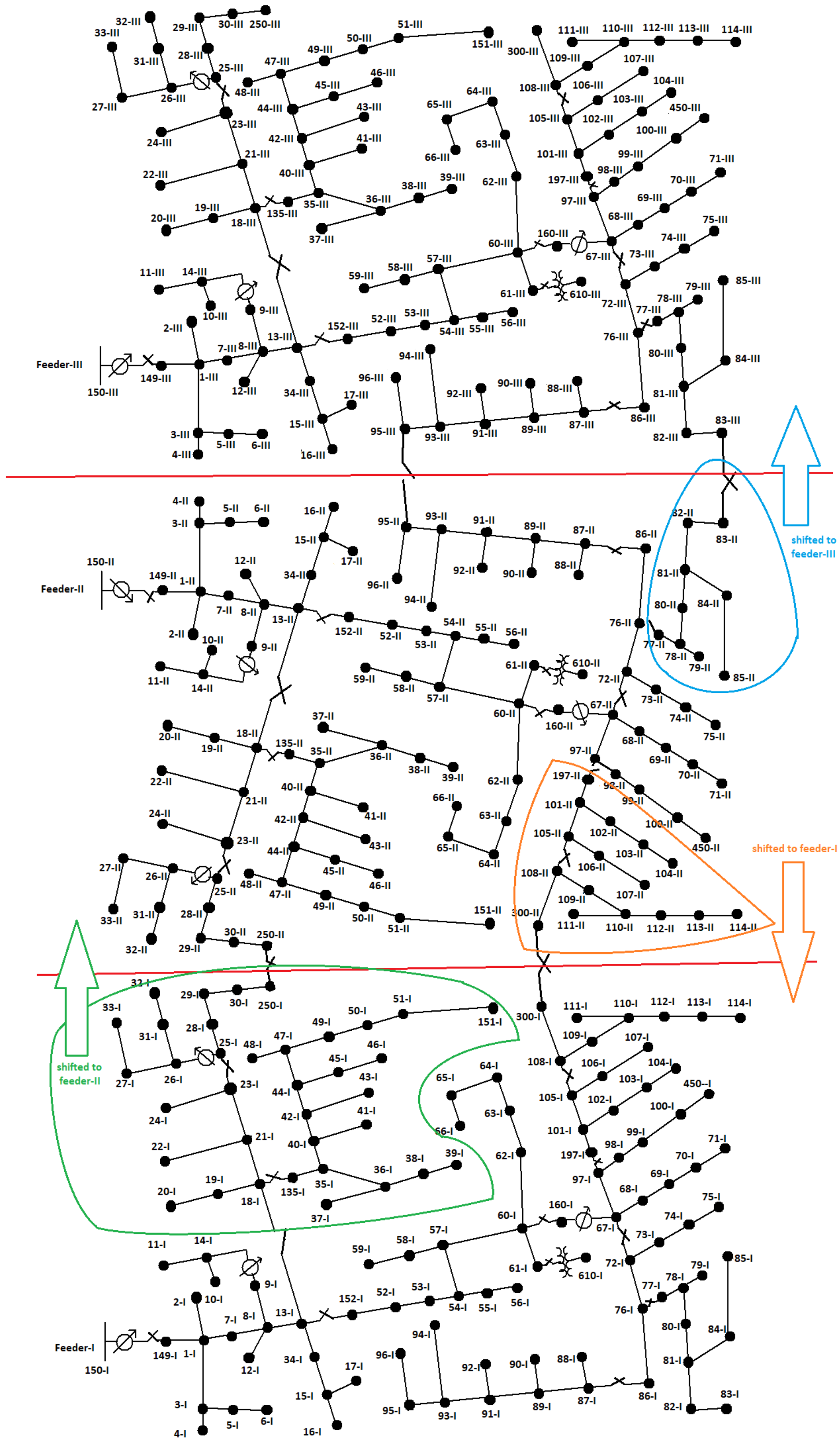

Figure 3.9: Solution of case 1 parallel structure

feeders without breaking the constraints. The objective value is 6 in this case. And the computing time is 0.1599s.

Case 2

Table 3.11 displays the settings of this case. And the solution of this case is illustrated in Figure

Table 3.11: Constraint Settings of Case 2 Parallel Structure System

P_{m1}/P.U	P_{m2}/P.U	P_{m3}/P.U	Q_{m1}/P.U	Q_{m2}/P.U	Q_{m3}/P.U	P_{cap}/P.U	Q_{cap}/P.U
6.432	8.8	9.5	3.616	5.1	5.5	10.67	6

3.10. In this case, we get a solution with objective value of 8. All the tie switches are closed and the switches (13-I, 18-I), (76-II, 77-II), (86-II, 87-II) and (97-II, 197-II) are open. This is the most complicated case of the parallel structure system with a maximum objective value. The computing time is 0.1532s in this case.

3.5.2 Triangle Structure System

Case 1

The constraints setting are shown in Table 3.12. The solution is shown as Figure 3.11. In this case,

Table 3.12: Constraint Settings of Case 1 Triangle Structure System

P_{m1}/P.U	P_{m2}/P.U	P_{m3}/P.U	Q_{m1}/P.U	Q_{m2}/P.U	Q_{m3}/P.U	P_{cap}/P.U	Q_{cap}/P.U
6	10	10	3.2	6	6	10.67	6

we get a solution with objective value of 6. The tie switches (250-I, 95-II), (250-II, 95-III) and (250-III, 95-I) are closed and sectionalizing switches (23-I, 25-I), (67-I, 72-I) and (86-III, 87-III) are open. Feeder-I shifts load in the green circle to Feeder-II and load in the blue circle to Feeder-

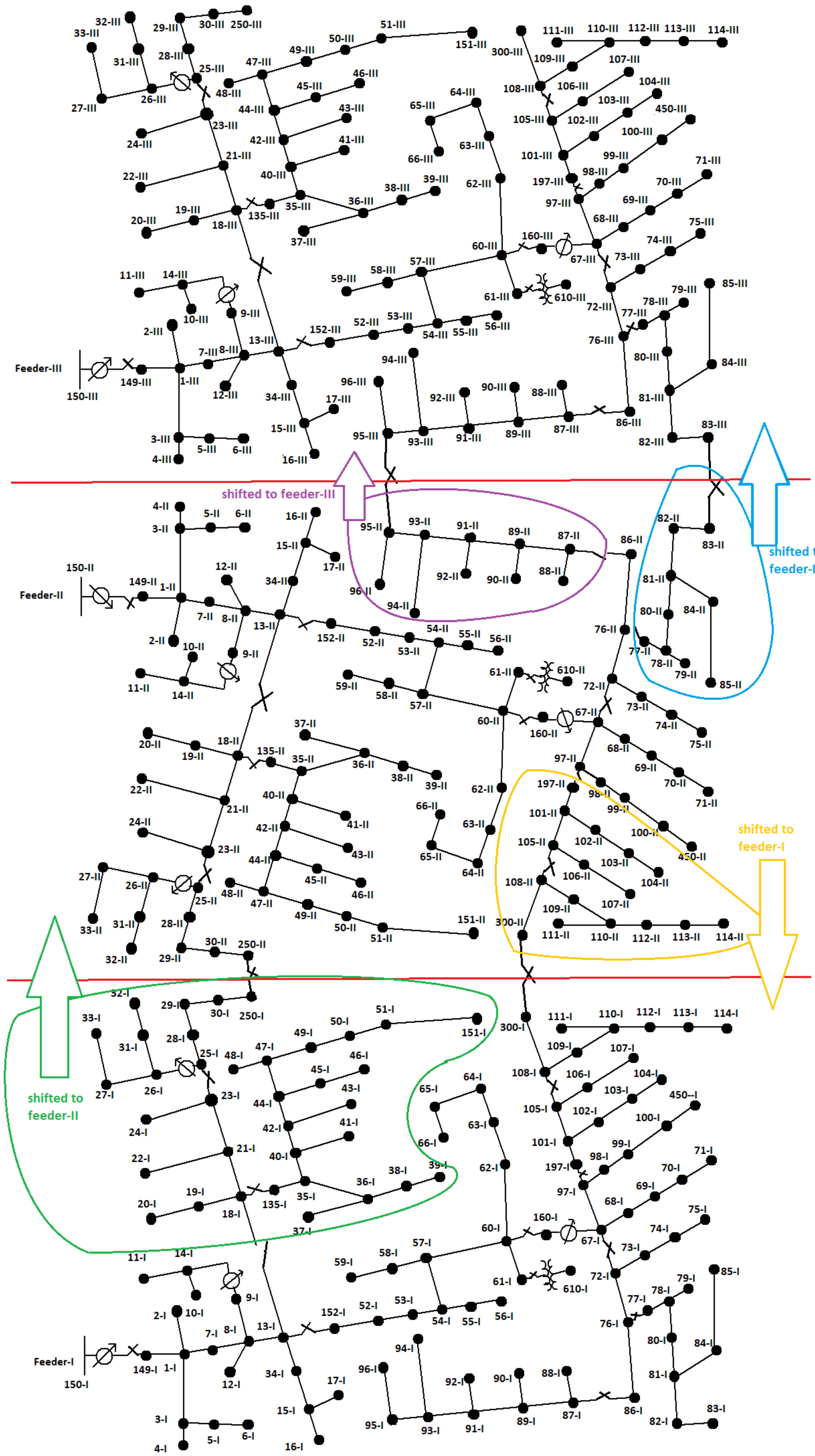

Figure 3.10: Solution of case 2 parallel structure

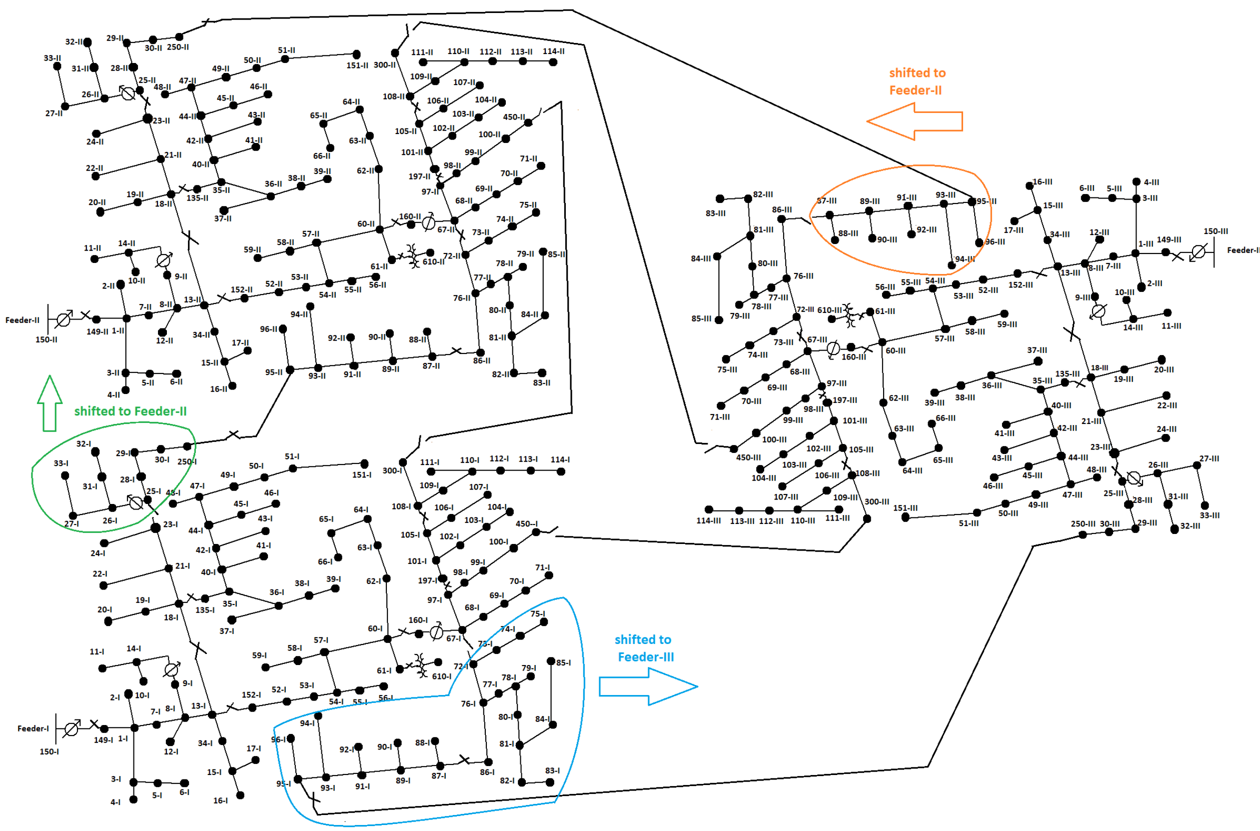

Figure 3.11: Solution of case 1 triangle structure

III. But it breaks the capacity of Feeder-III. So Feeder-III shifts the load in the orange circle to Feeder-II. The computing time is 0.2451s in this case.

Case 2

The constraints settings are demonstrated in Table 3.13. Figure 3.12 shows the solution of this

Table 3.13: Constraint Settings of Case 2 Triangle Structure System

P_{m1}/P.U	P_{m2}/P.U	P_{m3}/P.U	Q_{m1}/P.U	Q_{m2}/P.U	Q_{m3}/P.U	P_{cap}/P.U	Q_{cap}/P.U
5	9.5	10	3	5.4	6	10.67	6

case. This is a very complicated case. The tie switches (250-I, 95-II), (300-I, 450-II), (250-II, 95-III), (300-II, 450-III), (250-III, 95-I) are closed and the sectionalizing switches (23-I, 25-I), (67-I, 72-I), (97-I, 197-I), (105-II, 108-II) and (86-III, 87-III) are open. The objective value is 10 in this case. The computing time is 0.3074s in case 2.

3.5.3 Internal Reconfiguration Case

For the next step, we perform some modifications to the parallel structure network. Six more sectionalizing switches, (54-I, 94-I), (151-I, 300-I), (54-II, 94-II), (151-II, 300-II), (54-III, 94-III) and (151-III, 300-III) are added. But these six switches are open in normal operation. We carry out a test to find out if the internal structure could be changed during the reconfiguration process. The constraint settings are shown in Table 3.14. The solution is shown as Figure 3.13. The objective

Table 3.14: Constraint Settings of Internal Structure Reconfiguration

P_{m1}/P.U	P_{m2}/P.U	P_{m3}/P.U	Q_{m1}/P.U	Q_{m2}/P.U	Q_{m3}/P.U	P_{cap}/P.U	Q_{cap}/P.U
7	10	10	3.8	7	7	10.67	6

value is 6 in this case. The tie switches (250-I, 250-II) and (300-I, 300-II) are closed. The switches

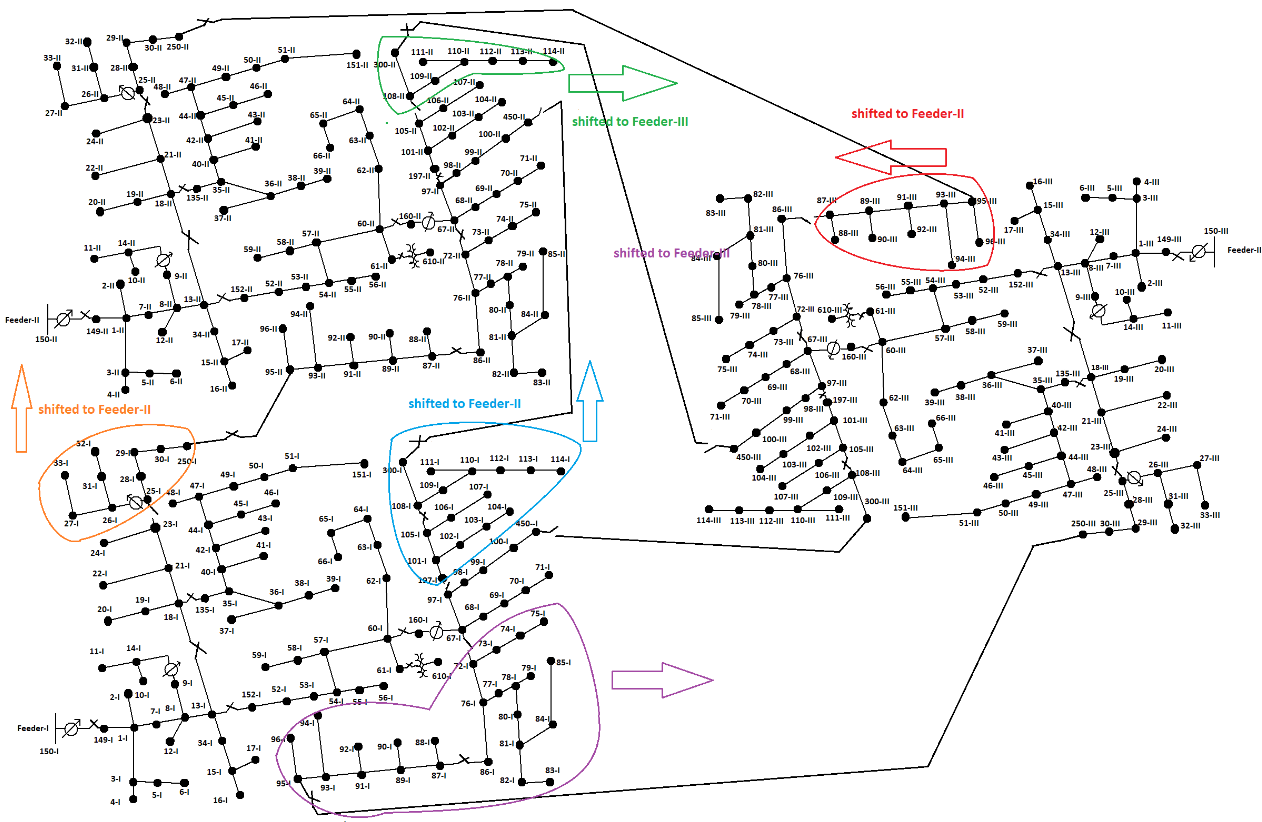

Figure 3.12: Solution of case 2 triangle structure

(23-I, 25-I), (67-I, 72-I) and (60-I, 160-I) are open and the switch (54-I, 94-I) is closed. By closing the switch (54-I, 94-I), we could change the internal structure of Feeder-I to keep the voltage constraints so that we are able to avoid shifting more loads to Feeder-II, which results in a higher objective value. The computing time is 0.2199s in this case.

3.6 Summary

With so many test cases above, the performance of our algorithm proves to be very good. Each feeder tends to shift loads at its leaf node to avoid unnecessary switch operation. And it is pretty fast to solve the problem. But the load shedding is not considered in our case, which could be studied in future work.

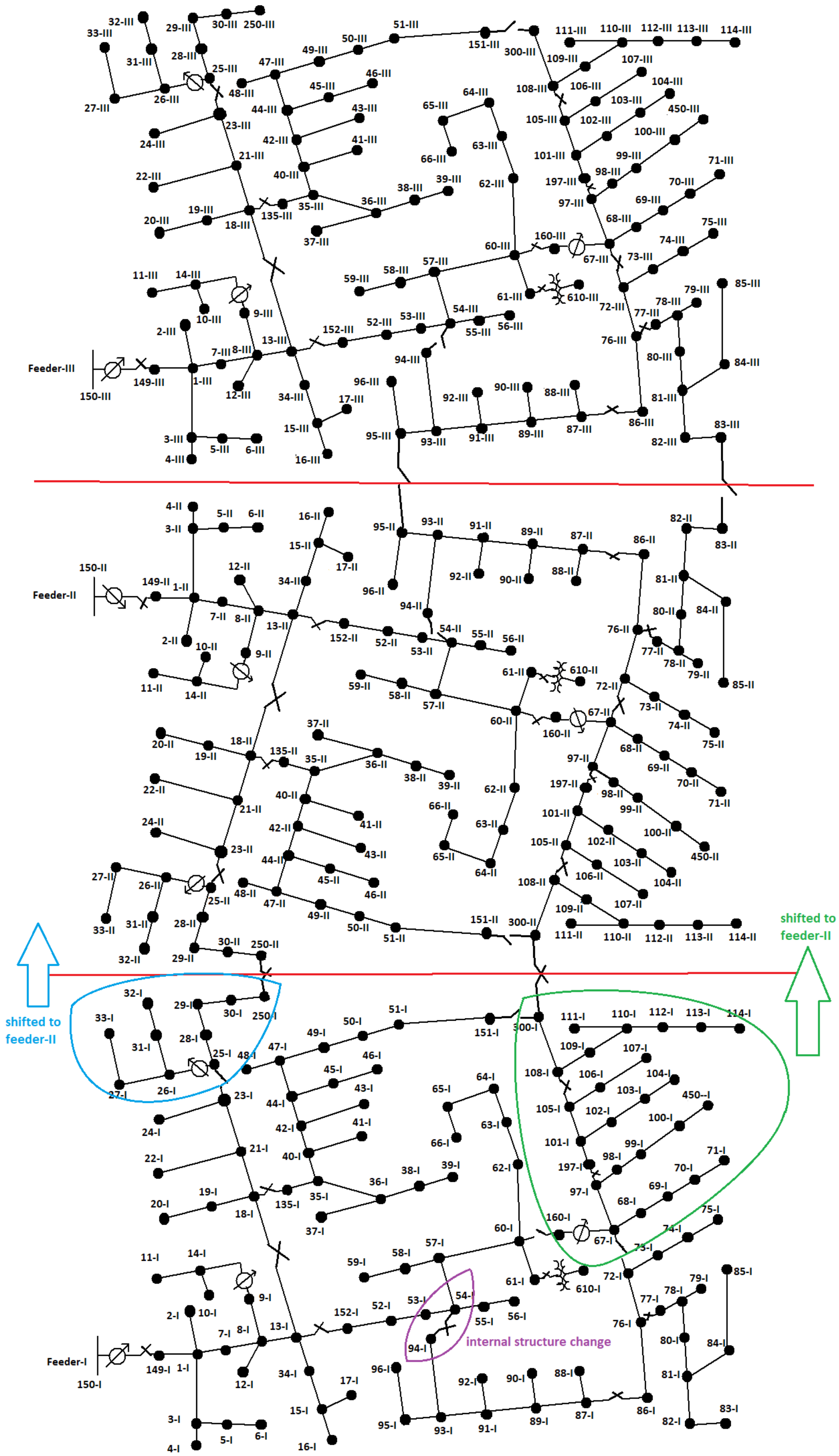

Figure 3.13: Solution of internal structure reconfiguration

Chapter 4

CONCLUSIONS

This thesis focuses on the load pickup problem and the feeder reconfiguration problem in the distribution network restoration process. For the load pickup problem, we formulate the problem as a multidimensional knapsack problem and solve it with Gurobi 8.1 solver in a Julia 1.0.2 environment. Also the $H^{1/(d+1)}$ approximation algorithm is proposed, which could get a not bad approximate solution with less time. Many test cases have been performed on 38-node, 123-node and 8500-node system to assess the performance of these two algorithms. Based on the case studies, we could find out that the original knapsack ILP algorithm could give out the accurate solution but it could take a long time to solve the problem. The approximation algorithm could solve the problem with much less time and the accuracy of the approximate solution is not worse than 50% of the accurate solution got by ILP. So the approximation algorithm does decrease the computing time. However, the performance of computing time of the ILP algorithm is not that bad. Therefore, whether it is necessary to use the approximation algorithm in our cases still needs further research.

In the feeder reconfiguration problem, we formulate it as an ILP problem as well. In order to do this, we think of an objective function, use big M method and introduce many variables to implement the functions of system constraints. By performing many tests on the three-feeder 16-node system, the accuracy of our algorithm is proved. Then we apply our algorithm to a larger scale distribution system, a three-feeder 375-node system, which is modified from IEEE 123-node test system. In the 375-node system case, the algorithm could make out whether there is a switch attached to one line and the solution given out by the algorithm is accurate even in some very complicated cases. What is more intriguing is that even the internal topological structure change is realized by our algorithm. In conclusion, our algorithm to solve the feeder reconfiguration problem has a very good performance and it could gives out the accurate solution with a very short computing time.

Chapter 5

FUTURE WORK

A plenty of future work could be done with this thesis. For the load pickup problem, firstly, further research needs to be done to show the accuracy guarantee of the approximation algorithm in a mathematical way. We can only show that the $H^{1/(d+1)}$ algorithm is a $1/(d+1)$-approximation algorithm but the actual performance is much better than this. In addition, we could try some more complicated distribution systems for case studies. For example, we could add DGs and reactive power compensators to the system so that the ILP algorithm could have a very long computing time, which could show the necessity of using approximation algorithm.

When it comes to the feeder reconfiguration problem, although the performance of our algorithm is very good, the load shedding is not considered in this thesis. We assume that the other feeders could supply the power to all the shifted loads from the faulted feeder. However, in a serious distribution system power outage, load shedding could be an usual way to protect the whole network. If the load shedding situation is considered in the algorithm, the formulation of this problem could be more complicated since we need more constraints and more variables to keep the functions of our constraints. Another aspect which needs further research is the expression of the capacity constraints. For both the feeder capacity constraints and the line capacity constraints, we all used active power and reactive power to denote the constraints. However, these constraints are characterized by apparent power and current in practice. For the next step, we could try to find out if we can use current and apparent power to formulate these constraints. And maybe we need to think of some new formulations for this problem since the apparent power and the current would bring in the quadratic term, making the linear programming unable to use.

BIBLIOGRAPHY

[1] Amir Abiri-Jahromi, Mahmud Fotuhi-Firuzabad, Masood Parvania, and Mohsen Mosleh. Optimized sectionalizing switch placement strategy in distribution systems. *IEEE Transactions on Power Delivery*, 27(1):362–370, 2012.

[2] Hamed Ahmadi and José R Martı. Linear current flow equations with application to distribution systems reconfiguration. *IEEE Transactions on Power Systems*, 30(4):2073–2080, 2015.

[3] Hamed Ahmadi and José R Martí. Mathematical representation of radiality constraint in distribution system reconfiguration problem. *International Journal of Electrical Power & Energy Systems*, 64:293–299, 2015.

[4] A Ajaja and Francisco D Galiana. Distribution network reconfiguration for loss reduction using milp. In *2012 IEEE PES Innovative Smart Grid Technologies (ISGT)*, pages 1–6. IEEE, 2012.

[5] Mesut E Baran and Felix F Wu. Optimal capacitor placement on radial distribution systems. *IEEE Transactions on power Delivery*, 4(1):725–734, 1989.

[6] Alberto Caprara, Hans Kellerer, and Ulrich Pferschy. A ptas for the multiple subset sum problem with different knapsack capacities. *Information Processing Letters*, 73(3-4):111–118, 2000.

[7] Alberto Caprara, Hans Kellerer, Ulrich Pferschy, and David Pisinger. Approximation algorithms for knapsack problems with cardinality constraints. *European Journal of Operational Research*, 123(2):333–345, 2000.

[8] Chao-Shun Chen, Chia-Hung Lin, and Hung-Ying Tsai. A rule-based expert system with colored petri net models for distribution system service restoration. *IEEE Transactions on Power Systems*, 17(4):1073–1080, 2002.

[9] Chen Chen, Jianhui Wang, Feng Qiu, and Dongbo Zhao. Resilient distribution system by microgrids formation after natural disasters. *IEEE Transactions on smart grid*, 7(2):958–966, 2016.

[10] Seyhan Civanlar, JJ Grainger, Ho Yin, and SSH Lee. Distribution feeder reconfiguration for loss reduction. *IEEE Transactions on Power Delivery*, 3(3):1217–1223, 1988.

[11] Debapriya Das. A fuzzy multiobjective approach for network reconfiguration of distribution systems. *IEEE transactions on power delivery*, 21(1):202–209, 2006.

[12] Pilar Meneses de Quevedo, Javier Contreras, Marcos J Rider, and Javad Allahdadian. Contingency assessment and network reconfiguration in distribution grids including wind power and energy storage. *IEEE Transactions on Sustainable Energy*, 6(4):1524–1533, 2015.

[13] W El-Khattam and MMA Salama. Impact of distributed generation on voltage profile in deregulated distribution system. In *Proceedings of the power systems 2002 conference, impact of distributed generation, Clemson, SC, USA*, pages 13–15. Citeseer, 2002.

[14] Alan M Frieze, MRB Clarke, et al. Approximation algorithms for the m-dimensional 0-1 knapsack problem: worst-case and probabilistic analyses. *European Journal of Operational Research*, 15(1):100–109, 1984.

[15] Vineet Goyal and R Ravi. A ptas for the chance-constrained knapsack problem with random item sizes. *Operations Research Letters*, 38(3):161–164, 2010.

[16] Y-Y Hsu, H-M Huang, H-C Kuo, SK Peng, CW Chang, KJ Chang, HS Yu, CE Chow, and RT Kuo. Distribution system service restoration using a heuristic search approach. *IEEE Transactions on Power Delivery*, 7(2):734–740, 1992.

[17] Rabih A Jabr, Ravindra Singh, and Bikash C Pal. Minimum loss network reconfiguration using mixed-integer convex programming. *IEEE Transactions on Power systems*, 27(2):1106–1115, 2012.

[18] Christos G Kaloudas, Luis F Ochoa, Ben Marshall, Shanti Majithia, and Ian Fletcher. Assessing the future trends of reactive power demand of distribution networks. *IEEE Transactions on Power Systems*, 32(6):4278–4288, 2017.

[19] Areg Karapetyan, Majid Khonji, Chi-Kin Chau, and Khaled Elbassioni. Online algorithm for demand response with inelastic demands and apparent power constraint. *arXiv preprint arXiv:1611.00559*, 2016.

[20] Hans Kellerer, Ulrich Pferschy, and David Pisinger. Knapsack problems. 2004, 2003.

[21] Ali Khodabakhsh, Ger Yang, Soumya Basu, Evdokia Nikolova, Michael C Caramanis, Thanasis Lianeas, and Emmanouil Pountourakis. A submodular approach for electricity distribution network reconfiguration. *arXiv preprint arXiv:1711.03517*, 2017.

[22] Majid Khonji, Chi-Kin Chau, and Khaled Elbassioni. Optimal power flow with inelastic demands for demand response in radial distribution networks. *IEEE Transactions on Control of Network Systems*, 5(1):513–524, 2018.

[23] Marina Lavorato, John F Franco, Marcos J Rider, and Rubén Romero. Imposing radiality constraints in distribution system optimization problems. *IEEE Transactions on Power Systems*, 27(1):172–180, 2012.

[24] Seung-Jae Lee, Seong-Il Lim, and Bokk-Shin Ahn. Service restoration of primary distribution systems based on fuzzy evaluation of multi-criteria. *IEEE Transactions on Power Systems*, 13(3):1156–1163, 1998.

[25] Na Li, Guannan Qu, and Munther Dahleh. Real-time decentralized voltage control in distribution networks. In *2014 52nd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 582–588. IEEE, 2014.

[26] Zongzhi Li. *Optimal Investment Decision-Making for Highway Transportation Asset Management under Risk and Uncertainty*. Midwest Regional University Transportation Center, University of Wisconsin , 2007.

[27] S-I Lim, S-J Lee, M-S Choi, D-J Lim, and B-N Ha. Service restoration methodology for multiple fault case in distribution systems. *IEEE Transactions on Power Systems*, 21(4):1638–1644, 2006.

[28] Whei-Min Lin and Hong-Chan Chin. A new approach for distribution feeder reconfiguration for loss reduction and service restoration. *IEEE Transactions on Power Delivery*, 13(3):870–875, 1998.

[29] C-C Liu, Seung Jae Lee, and SS Venkata. An expert system operational aid for restoration and loss reduction of distribution systems. *IEEE Transactions on Power Systems*, 3(2):619–626, 1988.

[30] Shanshan Ma, Shiyang Li, Zhaoyu Wang, Anmar Arif, and Kang Ma. A novel milp formulation for fault isolation and network reconfiguration in active distribution systems. In *2018 IEEE Power & Energy Society General Meeting (PESGM)*, pages 1–5. IEEE, 2018.

[31] José RS Mantovani, Fernando Casari, and Rubén A Romero. Reconfiguração de sistemas de distribuição radiais utilizando o critério de queda de tensão. *Controle and Automacao*, pages 150–159, 2000.

[32] J McCalley and E Gil. Data collection following katrina: Interdependencies across time, space, and subsystems characterizing bulk energy transportation. *NSF-ECS final report*, 2006.

[33] Tingru Meng, Meng Xu, Guibin Zou, Jingjing Yang, Jian Zhang, and Xia Lin. Voltage regulation based on hierarchical and district-dividing control for active distribution network. In *2015 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, pages 1–5. IEEE, 2015.

[34] Karen Nan Miu, Hsiao-Dong Chiang, Bentao Yuan, and Gary Darling. Fast service restoration for large-scale distribution systems with priority customers and constraints. *IEEE Transactions on Power Systems*, 13(3):789–795, 1998.

[35] B Moradzadeh and K Tomsovic. Mixed integer programming-based reconfiguration of a distribution system with battery storage. In *2012 North American Power Symposium (NAPS)*, pages 1–6. IEEE, 2012.

[36] NERC. Hurricane sandy event analysis report. `https://www.nerc.com/pa/rrm/ea/Oct2012HurricanSandyEvntAnlyssRprtDL/Hurricane_Sandy_EAR_20140312_Final.pdf`, 2014. [Accessed: April 16 2019].

[37] Cuong P Nguyen and Alexander J Flueck. Agent based restoration with distributed energy storage support in smart grids. *IEEE Transactions on Smart Grid*, 3(2):1029–1038, 2012.

[38] The City Univeristy of New York. Puerto rico one year after hurricane maria. `https://centropr.hunter.cuny.edu/sites/default/files/data_briefs/Hurricane_maria_1YR.pdf`, October 2018. [Accessed: April 17 2019].

[39] Executive Office of the President. Council of Economic Advisers. Economic benefits of increasing electric grid resilience to weather outages. `https://www.energy.gov/sites/prod/files/2013/08/f2/Grid%20Resiliency%20Report_FINAL.pdf`, 2013. [Accessed: April 17 2019].

[40] Ritu Parasher. Load flow analysis of radial distribution network using linear data structure. *arXiv preprint arXiv:1403.4702*, 2014.

[41] Mario VF Pereira and Leontina MVG Pinto. Application of sensitivity analysis of load supplying capability to interactive transmission expansion planning. *IEEE Transactions on Power Apparatus and Systems*, (2):381–389, 1985.

[42] IEEE PES. Ieee pes amps dsas test feeder working group resources. `http://sites.ieee.org/pes-testfeeders/resources/`, 1992. [Accessed: April 29 2019].

[43] IEEE PES. Ieee pes amps dsas test feeder working group resources. `http://sites.ieee.org/pes-testfeeders/resources/`, 2010. [Accessed: April 29 2019].

[44] Shiva Poudel and Anamika Dubey. Critical load restoration using distributed energy resources for resilient power distribution system. *IEEE Transactions on Power Systems*, 34(1):52–63, 2019.

[45] R Rajaram, K Sathish Kumar, and N Rajasekar. Power system reconfiguration in a radial distribution network for reducing losses and to improve voltage profile using modified plant growth simulation algorithm with distributed generation (dg). *Energy Reports*, 1:116–122, 2015.

[46] Hua Shao, Yujie Shi, Jianpu Yuan, Jiakun An, and Jianhua Yang. Analysis on voltage profile of distribution network with distributed generation. In *IOP Conference Series: Earth and Environmental Science*, volume 113, page 012170. IOP Publishing, 2018.

[47] Dariush Shirmohammadi. Service restoration in distribution networks via network reconfiguration. In *Proceedings of the 1991 IEEE Power Engineering Society Transmission and Distribution Conference*, pages 626–632. IEEE, 1991.

[48] Jignesh M Solanki, Sarika Khushalani, and Noel N Schulz. A multi-agent solution to distribution systems restoration. *IEEE Transactions on Power systems*, 22(3):1026–1034, 2007.

[49] Ching-Tzong Su and Chu-Sheng Lee. Network reconfiguration of distribution systems using improved mixed-integer hybrid differential evolution. *IEEE Transactions on Power Delivery*, 18(3):1022–1027, 2003.

[50] Tim Taylor and David Lubkeman. Implementation of heuristic search strategies for distribution feeder reconfiguration. *IEEE Transactions on Power Delivery*, 5(1):239–246, 1990.

[51] Yezhou Wang, Chen Chen, Jianhui Wang, and Ross Baldick. Research on resilience of power systems under natural disastersa review. *IEEE Transactions on Power Systems*, 31(2):1604–1613, 2016.

[52] Arif Wazir and Naeem Arbab. Analysis and optimization of ieee 33 bus radial distributed system using optimization algorithm. *JETAE) J. Emerg. Trends Appl. Eng*, 1(2):2518–4059, 2016.

[53] Justin C Williams. A linear-size zeroone programming model for the minimum spanning tree problem in planar graphs. *Networks: An International Journal*, 39(1):53–60, 2002.

[54] H Lee Willis. *Power distribution planning reference book*. CRC press, 1997.

[55] Chen Yuan, Mahesh S Illindala, and Amrit S Khalsa. Modified viterbi algorithm based distribution system restoration strategy for grid resiliency. *IEEE Transactions on Power Delivery*, 32(1):310–319, 2017.

[56] Dong Zhang, Zhengcai Fu, and Liuchun Zhang. An improved ts algorithm for loss-minimum reconfiguration in large-scale distribution systems. *Electric Power Systems Research*, 77(5-6):685–694, 2007.

[57] H Zheng, Y Cheng, B Gou, D Frank, A Bern, and WE Muston. Impact of automatic switches on power distribution system reliability. *Electric Power Systems Research*, 83(1):51–57, 2012.

[58] Qin Zhou, Dariush Shirmohammadi, and W-HE Liu. Distribution feeder reconfiguration for service restoration and load balancing. *IEEE Transactions on Power Systems*, 12(2):724–729, 1997.

[59] Hao Zhu and Hao Jan Liu. Fast local voltage control under limited reactive power: Optimality and stability analysis. *IEEE Transactions on Power Systems*, 31(5):3794–3803, 2016.